

Chapter 10

Hypothesis Tests for One Population Mean

Sections 10.1 and 10.3 Hypothesis Testing

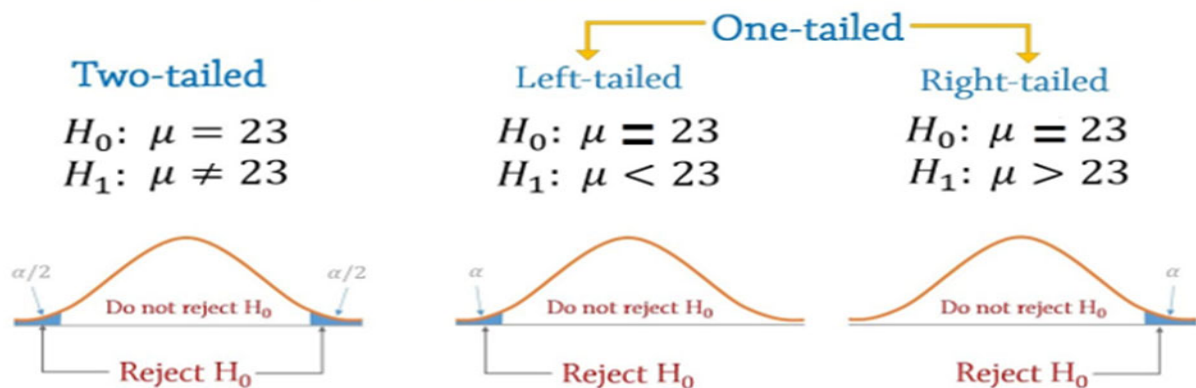
Hypothesis Testing is an attempted proof by statistical contradiction where the preconceived idea about the value of a parameter is called a hypothesis. It's called the hypothesized value for the population mean μ and it's denoted by μ_0 .

General Statements:

- (1) The hypothesis to be tested is called the null hypothesis and is denoted by H_0 . The null hypothesis is always tested against some ALTERNATIVE HYPOTHESIS which always the opposite of H_0 and it's denoted by H_1 .

$$H_0 : \mu = \mu_0 \quad \text{vs} \quad H_1 : \begin{cases} 1. \mu \neq \mu_0 \\ 2. \mu < \mu_0 \\ 3. \mu > \mu_0 \end{cases}$$

Hypothesis Testing



- (2) The null hypothesis, H_0 , is rejected whenever the sample mean, \bar{X} , is far enough away from the hypothesized value, μ_0 . We will discuss how far enough later. If we reject the null hypothesis, then we have enough evidence to accept the alternative hypothesis, H_1 . When do we fail to reject the null hypothesis, H_0 ? When the sample mean, \bar{X} , is NOT far enough away from the hypothesized value, μ_0 . This does not necessarily mean that the null hypothesis is true, it just means that based on how far the sample mean, \bar{X} , is away from the hypothesized mean, μ_0 , we cannot reject H_0 .
- (3) The probability of making a Type I error is called the **α level of significance**. In our testing, we will be given a significance level, α . If not by default $\alpha = .05$.

Procedure of Hypothesis Testing Using the TI 83/84:

Assume that the null hypothesis is true. $H_0: \mu = \mu_0$

Calculate the p-value. P-value = $P(\text{Observing } \bar{X} \mid H_0 \text{ is true})$. (Using the TI-83/84)

Reject H_0 : If the p-value is less than the alpha level of significance ($\text{p-value} < \alpha$) then we reject H_0 and accept the H_1 .

Fail to Reject H_0 : If the p-value is greater or equal to the alpha level of significance ($\text{p-value} \geq \alpha$) then we fail to reject H_0 ; i.e. we accept the H_0 .

Type I and Type II Errors

There are two types of incorrect decisions.

Type I Error: Rejecting the null hypothesis when it is true.

Type II Error: Not rejecting the null hypothesis when it is false.

Example 1. Proline fishing line has a breaking strength that is normally distributed with a mean of 30 pounds. The engineers have developed a new, less expensive manufacturing process, but are worried that the mean breaking strength may be less than 30 pounds. They assume that the breaking strength is unchanged, unless there is convincing evidence to the contrary. A sample of 25 of these new lines was tested and their average breaking strength came out to 28.4 pounds and a standard deviation of 3. Is there compelling evidence that the new lines are weaker? Use 0.05 as the level of significance.

Soln: Step1: State the null and alternative hypotheses.

$$H_0: \mu = 30 \quad \text{vs} \quad H_1: \mu < 30$$

Step2: Write the given information using proper symbols.

$$n=25, \bar{x} = 28.4, S = 3, \bar{X} \sim t_{24}$$

$$\text{Test statistic. } t_{24} = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = -2.67 \quad (\text{From TI-83/84})$$

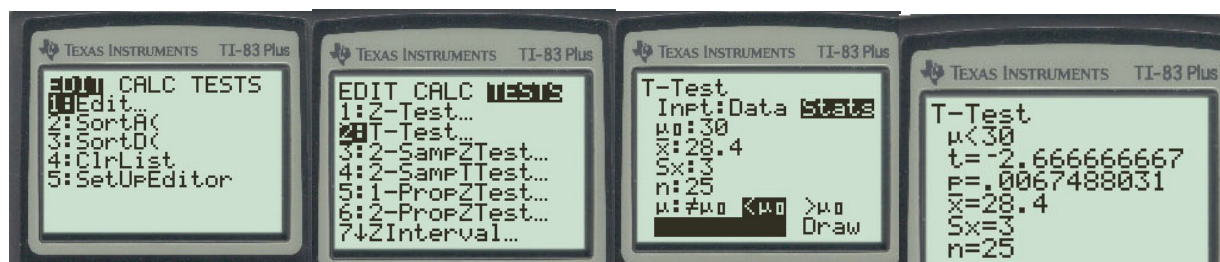
Step3: Find the P-value. If the **P-value is less than α** , Reject H_0 .

$$\text{P-value} = .0067 \quad (\text{From TI-83/84})$$

Step4: Conclusion. Since the P-value=.0067 is less than $\alpha=.05$, we reject H_0 and claim there is sufficient evidence to conclude the population mean μ per order is less than 30 pounds.

Note: Interpret the P-Value=0.0067. There is a 0.0067 probability of obtaining a sample mean $\bar{X}=28.4$ or less from a population whose mean is $\mu=30$ pounds.

Using TI-83/84: stat, TESTS, T-TEST (is #2), then choose “stats”. Enter μ_0 : 30, \bar{X} : 28.4, S_x : 3, n: 25, μ : $< \mu_0$, put your cursor on the appropriate Test and hit “enter”, put your cursor on “calculate” and hit enter. Look for the P-Value: p=.0067488031. Also note the Test Statistic for Step #2 comes from the calculator even though we are not going to use it. It’s the T-Score telling us how far $\bar{X} = 28.4$ is away from the Hypothesized mean, $\mu_0 = 30$, $t = -2.67$. (see calculator pictures below)



Example 2. In justifying their demands for higher wages, the employees in the shipping department of a large mail order house report that, on the average, the department completes an order within 13 minutes. The general manager took a random sample of 400 orders and found the average completion time is 14 minutes with a standard deviation of 10 minutes. If the general manager would like to prove to the employees that the order takes more minutes on the average, do the appropriate hypothesis testing. Use 0.03 as the level of significance.

Soln: Step1: State the null and alternative hypotheses.

$$H_0: \mu = 13 \quad \text{vs} \quad H_1: \mu > 13$$

Step2: Write the given information using proper symbols.

$$n=400, \bar{x} = 14, S = 10, \bar{X} \sim t_{399}$$

$$\text{Test statistic. } t_{399} = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = 2 \quad (\text{From TI-83})$$

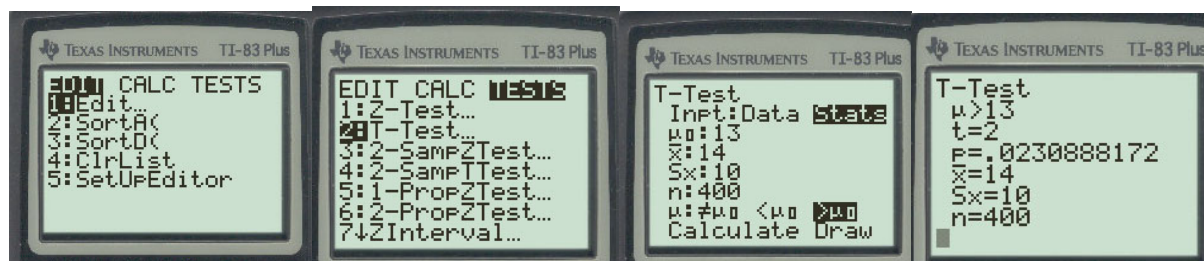
Step3: If the **P-value** is **less than α** , Reject H_0 . P-value=.023 (From TI-83/84)

Step4: Conclusion. Since the P-value=.023 is less than $\alpha=.03$, we reject H_0 and claim there is sufficient evidence to conclude the population mean μ per order is greater than 13 minutes.

Note: Interpret the P-Value=0.023. There is a 0.023 probability of obtaining a sample mean $\bar{X}=14$ minutes or greater from a population whose mean is $\mu=13$ minutes.

Using TI-83/84: stat, TESTS, T-TEST (is #2), then choose “stats”.

Enter μ_0 : 13, \bar{X} : 14, S_x : 10, n: 400, μ : $> \mu_0$, put your cursor on the appropriate Test and hit “enter”, put your cursor on “calculate” and hit enter. Look for the P-Value: $p=.0230888172$. (see calculator pictures below)



Example 3: A California wine producer states that the volume of wine in its standard size bottles averages 750 ml. An independent agency examines a random sample of 17 bottles and finds the average volume for the sample to be 721 ml with a sd of 48 ml. Is the average volume per bottle less than 750 ml? Use $\alpha = 0.01$. Assume that the volume of wine is normally distributed.

Soln. Step1: State the null and alternative hypotheses.

$$H_0: \mu = 750 \quad \text{vs} \quad H_1: \mu < 750$$

Step2: Write the given information using proper symbols.

$$n = 17, \bar{x} = 721, s = 48, \alpha = 0.01, \bar{X} \sim t_{16}$$

Find the test statistic. $t_{16} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$. $t_{16} = -2.491$ (From TI-83/84)

Step3: Find the P-value. If the **P-value** is **less than α** , Reject H_0 .

$$P\text{-value} = .0121 \quad (\text{From TI-83})$$

Step4: Conclusion. Since the P-value=.0121 is greater than $\alpha=.01$, we Do Not reject H_0 and claim there is NOT sufficient evidence to conclude the population mean μ is less than 750 ml.

Note: Interpret the P-Value=0.0121. There is a 0.0121 probability of obtaining a sample mean $\bar{X}=721$ ounces or less from a population whose mean is $\mu=750$ ounces.

Example 4: A sample of 10 cracker boxes yielded the following net weights (in ounces): 16.1, 16.01, 15.82, 15.93, 16.05, 16.05, 15.86, 15.90, 16.05, 16.13. Is there enough evidence to conclude the population mean is not 15.7 ounces? Assume that the net weight of the crackers is normally distributed.

Soln. Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: \mu = 15.7 \quad \text{vs} \quad \mathbf{H}_1: \mu \neq 15.7$$

Step2: Write the given information using proper symbols.

$$n = 10, \bar{x} = 15.99, s = 0.1056, \alpha = 0.01, \bar{x} \sim t_9$$

Find the test statistic. $t_9 = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$. $t_9 = 8.68265$ (From TI-83/84)

Step3: Find the P-value. If the **P-value is less than α** , Reject \mathbf{H}_0 .

$$\mathbf{P\text{-value}} = 1.1441254 \times 10^{-5} = 0.00001 \quad (\text{From TI-83/84})$$

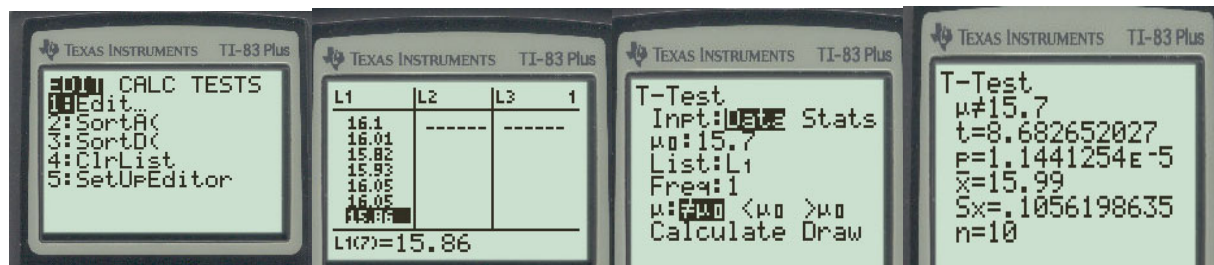
Step4: Conclusion. Since the P-value=.00001 is less than $\alpha=.01$, we reject \mathbf{H}_0 and claim there is sufficient evidence to conclude the population mean μ is NOT equal to 15 ounces.

Note: Interpret the P-Value=0.00001. There is a 0.00001 probability of obtaining a sample mean $\bar{X}=15.99$ ounces from a population whose mean is $\mu=15.7$ ounces.

In this example we must enter the data in a list first. **Stat, 1:Edit** and then enter the data in L1. Now follow the sequence of commands below.

Using TI-83/84: stat, TESTS, T-TEST (is #2), then choose “data”.

List: L1(enter the data in L1), Freq: 1, $\mu: \neq \mu_0$ (put your cursor on the appropriate Test and hit “enter”), put your cursor on “calculate” and hit enter. Note: Freq: 1 always. (see calculator pictures below)



Homework-Sections 10.1 and 10.3 Online - MyStatLab