

Student: _____
Date: _____

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Course: Math1111-Summer2018

Assignment: Section 4.4 Homework

1. Solve for x.

$$4^{2x-2} = 16$$

$$4^{2x-2} = 4^2 \Rightarrow 2x-2 = 2 \Rightarrow 2x = 4 \Rightarrow x = 2$$

The solution set is { 2 }.

2. Solve the following exponential equation by expressing each side as a power of the same base and then equating exponents.

$$4^{x+7} = 16^{x-4}$$

$$4^{x+7} = (4^2)^{x-4} \Rightarrow 4^{x+7} = 4^{2(x-4)} \Rightarrow x+7 = 2(x-4)$$
$$x+7 = 2x-8$$

The solution set is { 15 }.

$$x = 15$$

3. Solve the following exponential equation. Express the solution set in terms of natural logarithms. Then use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

$$9^x = 26$$

$$9^x = 26 \Rightarrow \ln 9^x = \ln 26 \Rightarrow x \cdot \ln 9 = \ln 26$$

What is the solution in terms of natural logarithms?

$$\Rightarrow x = \frac{\ln 26}{\ln 9} = 1.482823$$

The solution set is { $\frac{\ln 26}{\ln 9}$ }.

(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

What is the decimal approximation for the solution?

The solution set is { 1.48 }.

(Use a comma to separate answers as needed. Round to two decimal places as needed.)

4. Solve the following exponential equation by taking the natural logarithm on both sides. Express the solution in terms of natural logarithms. Then, use a calculator to obtain a decimal approximation for the solution.

$$2e^{7x} = 574$$

$$2e^{7x} = 574 \Rightarrow e^{7x} = \frac{574}{2} \Rightarrow e^{7x} = 287 \Rightarrow \ln e^{7x} = \ln 287$$

What is the solution in terms of natural logarithms?

$$\Rightarrow 7x = \ln 287 \Rightarrow x = \frac{\ln 287}{7}$$

The solution set is { $\frac{\ln 287}{7}$ }.

(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

$$x = 0.808497$$

What is the decimal approximation for the solution?

The solution set is { 0.81 }.

(Use a comma to separate answers as needed. Round to two decimal places as needed.)

5. Solve the exponential equation. Express the solution set in terms of natural logarithms. Then use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

$$e^{2x} - 7e^x + 6 = 0$$

$$\text{let } y = e^x \Rightarrow (e^x)^2 - 7(e^x) + 6 = 0 \Rightarrow y^2 - 7y + 6 = 0$$

What is the solution in terms of natural logarithms?

$$\Rightarrow (y-6)(y-1) = 0 \Rightarrow y=6 \text{ and } y=1$$

The solution set is $\{\ln 6, 0\}$.

(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

$$\text{if } y=6 \text{ then } e^x=6 \Rightarrow \ln e^x = \ln 6$$

$$\Rightarrow \boxed{x = \ln 6}$$

What is the decimal approximation for the solution?

The solution set is $\{1.79, 0\}$.

(Use a comma to separate answers as needed. Round to two decimal places as needed.)

$$\text{if } y=1 \text{ then } e^x=1 \Rightarrow \ln e^x = \ln 1$$

$$x = \ln 1$$

$$\boxed{x = 0}$$

6. Solve the following logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expression. Give the exact answer.

$$\log_2(x+7) = 3$$

$$2^3 = x+7 \Rightarrow 8 = x+7 \Rightarrow \boxed{x=1}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is $\{1\}$. (Type an integer or a simplified fraction.)

B. There is no solution.

7. Solve the following logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expression. Give the exact answer.

$$\log_2(x+23) = 5$$

$$2^5 = x+23 \Rightarrow 32 = x+23 \Rightarrow x=9$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is $\{9\}$. (Type an integer or a simplified fraction.)

B. There is no solution.

8. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expression.

$$6 \ln(6x) = 12 \Rightarrow \ln(6x) = \frac{12}{6} = 2 \Rightarrow \ln(6x) = 2 \Rightarrow e^{\ln(6x)} = e^2$$

$$6 \ln(6x) = 12$$

What is the exact solution? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\left\{ \frac{e^2}{6} \right\}$.
(Type an exact answer in simplified form.)
- B. There are infinitely many solutions.
- C. There is no solution.

$$\Rightarrow 6x = e^2$$

$$\Rightarrow x = \frac{e^2}{6}$$

What is the decimal approximation to the solution? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\{ 1.23 \}$.
(Type an integer or decimal rounded to two decimal places as needed.)
- B. There are infinitely many solutions.
- C. There is no solution.

$$\frac{e^2}{6} = 1.231509$$

9. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give an exact answer.

$$\log_{13} x + \log_{13}(12x-1) = 1 \Rightarrow \log_{13}(x(12x-1)) = 1 \Rightarrow 13^1 = x(12x-1) \Rightarrow 13 = 12x^2 - 12x$$

$$\Rightarrow 12x^2 - 12x - 13$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\left\{ \frac{13}{12} \right\}$. (Type an exact answer in simplified form.)
- B. There is no solution.

$$(12x-13)(x+1) = 0$$

$$12x-13=0 \Rightarrow x = \frac{13}{12}$$

$$x+1=0 \Rightarrow x = -1$$

10. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log_4(x+5) - \log_4(x-10) = 2$$

$$\log_4\left(\frac{x+5}{x-10}\right) = 2 \Rightarrow 4^2 = \frac{x+5}{x-10}$$

$$16(x-10) = x+5$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\{ 11 \}$.
(Simplify your answer. Use a comma to separate answers as needed.)
- B. There is no solution.

$$16x - 160 = x + 5$$

$$15x = 165 \Rightarrow x = \frac{165}{15} = 11$$

$x = -1$ is not in the domain of the log.

11. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log(x+7) - \log 2 = \log(4x+1)$$

$$\log\left(\frac{x+7}{2}\right) = \log(4x+1) \Rightarrow \frac{x+7}{2} = 4x+1$$

$$\Rightarrow x+7 = 2(4x+1)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is $\left\{ \frac{5}{7} \right\}$.
(Simplify your answer. Use a comma to separate answers as needed.)
- B. There is no solution.

$$x+7 = 8x+2$$

$$\Rightarrow 7x = 5$$

$$\Rightarrow x = \frac{5}{7}$$

12. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\ln(x-4) + \ln(x+3) = \ln(x-13)$$

$$\ln((x-4)(x+3)) = \ln(x-13)$$

$$(x-4)(x+3) = x-13 \Rightarrow x^2 - x - 12 = x - 13$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$$\Rightarrow x^2 - 2x + 13 = 0$$

$$(x-1)^2 = 0 \Rightarrow x = 1$$

$x = 1$ will make the log negative.

A. The solution set is { }.

(Simplify your answer. Use a comma to separate answers as needed.)

B. There is no solution.

13. The formula $A = 15.7 e^{0.0433t}$ models the population of a US state, A , in millions, t years after 2000.

a. What was the population of the state in 2000?

b. When will the population of the state reach 18.7 million?

$$A = 15.7 \cdot e^{0.0433(0)} = 15.7$$

a. In 2000, the population of the state was 15.7 million.

b. The population of the state will reach 18.7 million in the year 2004.
(Round down to the nearest year.)

$$18.7 = 15.7 e^{0.0433(t)}$$

$$\Rightarrow \frac{18.7}{15.7} = e^{0.0433t}$$

$$\ln\left(\frac{18.7}{15.7}\right) = 0.0433t$$

$$\Rightarrow t = \frac{\ln\left(\frac{18.7}{15.7}\right)}{0.0433}$$

$$t = 4.1038 \approx 4$$

14. Complete the table for a savings account subject to 4 compoundings yearly.

$$\left[A = P \left(1 + \frac{r}{n} \right)^{nt} \right]$$

Amount Invested	Number of Compounding Periods	Annual Interest Rate	Accumulated Amount	Time t In Years
\$12,000	4	6%	\$25,000	?

Let A represent the accumulated amount, P the amount invested, n the number of compounding periods, r the annual interest rate, and t the time. Find the time, t .

$$t = \underline{12.3} \text{ years}$$

(Do not round until the final answer. Then round to one decimal place as needed.)

$$25000 = 12000 \left(1 + \frac{0.06}{4} \right)^{4t}$$

$$\Rightarrow \ln\left(\frac{25000}{12000}\right) = \ln(1.015)^{4t}$$

$$\Rightarrow \ln\left(\frac{25000}{12000}\right) = 4t \ln(1.015)$$

$$\Rightarrow t = \frac{\ln\left(\frac{25000}{12000}\right)}{4 \cdot \ln(1.015)}$$

15. Complete the table for a savings account subject to continuous compounding.

$$(A = P e^{rt})$$

Amount Invested	Annual Interest Rate	Accumulated Amount	Time t in years
\$6000	3%	\$12,000	?

Let A represent the accumulated amount, P the amount invested, r the annual interest rate, and t the time. Find the time, t .

$$t \approx \underline{23.1} \text{ years}$$

(Round to one decimal place as needed.)

$$12000 = 6000 e^{0.03t} \Rightarrow \frac{12000}{6000} = e^{0.03t}$$

$$\Rightarrow 2 = e^{0.03t} \Rightarrow \ln 2 = \ln e^{0.03t}$$

$$\Rightarrow \ln 2 = 0.03t \Rightarrow t = \frac{\ln 2}{0.03} = 23.1$$

$$t = 23.1$$

1. 2

2. 15

3. $\frac{\ln 26}{\ln 9}$

1.48

4. $\frac{\ln 287}{7}$

0.81

5. $\ln 6,0$

1.79,0

6. A. The solution set is $\{ \underline{1} \}$. (Type an integer or a simplified fraction.)

7. A. The solution set is $\{ \underline{9} \}$. (Type an integer or a simplified fraction.)

8. A. The solution set is $\left\{ \frac{e^2}{6} \right\}$. (Type an exact answer in simplified form.)

A. The solution set is $\{ \underline{1.23} \}$. (Type an integer or decimal rounded to two decimal places as needed.)

9. A. The solution set is $\left\{ \frac{13}{12} \right\}$. (Type an exact answer in simplified form.)

10. A. The solution set is $\{ \underline{11} \}$. (Simplify your answer. Use a comma to separate answers as needed.)

11. A. The solution set is $\left\{ \frac{5}{7} \right\}$. (Simplify your answer. Use a comma to separate answers as needed.)

12. B. There is no solution.

13. 15.7

2004

14. 12.3
