Student: ____________________ **Date:** ____________________	**Instructor:** Andreas Lazari **Course:** Math2620 F - Fall 2018	**Assignment:** Chapter 8.1-Homework

1. Is the statement below true or false?

   The distribution of the sample mean, $\bar{x}$, will be normally distributed if the sample is obtained from a population that is normally distributed, regardless of the sample size.

   Choose the correct answer below.

   - ● True
   - ○ False

2. Determine whether the following statement is true or false.

   To cut the standard error of the mean in half, the sample size must be doubled.

   Choose the correct answer below.

   - ○ **A.** False. The sample size does not influence the value of the standard error.
   - ○ **B.** False. The sample size must be reduced by a factor of two to cut the standard error in half.
   - ● **C.** False. The sample size must be increased by a factor of four to cut the standard error in half.
   - ○ **D.** True.

3. A simple random sample of size $n = 62$ is obtained from a population with $\mu = 65$ and $\sigma = 6$. Does the population need to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed? Why? What is the sampling distribution of $\bar{x}$?

Does the population need to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed? Why?

- ● A. No because the Central Limit Theorem states that regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases.
- ○ B. Yes because the Central Limit Theorem states that only for underlying populations that are normal is the shape of the sampling distribution of $\bar{x}$ normal, regardless of the sample size, n.
- ○ C. No because the Central Limit Theorem states that only if the shape of the underlying population is normal or uniform does the sampling distribution of $\bar{x}$ become approximately normal as the sample size, n, increases.
- ○ D. Yes because the Central Limit Theorem states that the sampling variability of nonnormal populations will increase as the sample size increases.

What is the sampling distribution of $\bar{x}$? Select the correct choice below and fill in the answer boxes within your choice. (Type integers or decimals rounded to three decimal places as needed.)

$\mu_{\bar{x}} = \mu$

- ● A. The sampling distribution of $\bar{x}$ is normal or approximately normal with $\mu_{\bar{x}} =$ 65 and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{62}} = 0.762$.
- ○ B. ~~The sampling distribution of $\bar{x}$ is uniform~~ with $\mu_{\bar{x}} =$ ______ and $\sigma_{\bar{x}} =$ ______.
- ○ C. The sampling distribution of $\bar{x}$ follows Student's t-distribution with $\mu_{\bar{x}} =$ ______ and $\sigma_{\bar{x}} =$ ______.
- ○ D. The sampling distribution of $\bar{x}$ is skewed left with $\mu_{\bar{x}} =$ ______ and $\sigma_{\bar{x}} =$ ______.

4. Determine $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ from the given parameters of the population and sample size.

$\mu = 78$, $\sigma = 28$, $n = 49$

$\mu_{\bar{x}} = \mu \Rightarrow \mu_{\bar{x}} = 78$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \Rightarrow \sigma_{\bar{x}} = \frac{28}{\sqrt{49}} = \frac{28}{7} = 4.$

$\mu_{\bar{x}} =$ 78

$\sigma_{\bar{x}} =$ 4.

5. Determine $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ from the given parameters of the population and sample size.

$\mu = 40$, $\sigma = 13$, $n = 30$

$\sigma_{\bar{x}} = \frac{13}{\sqrt{30}} = 2.373$

$\mu_{\bar{x}} =$ 40

$\sigma_{\bar{x}} =$ 2.373

(Round to three decimal places as needed.)

6. Suppose a simple random sample of size n = 10 is obtained from a population with $\mu = 66$ and $\sigma = 14$.
(a) What must be true regarding the distribution of the population in order to use the normal model to compute probabilities regarding the sample mean? Assuming the normal model can be used, describe the sampling distribution $\bar{x}$.
(b) Assuming the normal model can be used, determine $P(\bar{x} < 69.4)$.
(c) Assuming the normal model can be used, determine $P(\bar{x} \geq 67.9)$.

(a) What must be true regarding the distribution of the population?

- A. The population must be normally distributed and the sample size must be large.
- ● B. The population must be normally distributed.
- C. There are no requirements on the shape of the distribution of the population.
- D. Since the sample size is large enough, the population distribution does not need to be normal.

Assuming the normal model can be used, describe the sampling distribution $\bar{x}$.

- A. Normal, with $\mu_{\bar{x}} = 66$ and $\sigma_{\bar{x}} = \frac{10}{\sqrt{14}}$
- ● B. Normal, with $\mu_{\bar{x}} = 66$ and $\sigma_{\bar{x}} = \frac{14}{\sqrt{10}}$
- C. Normal, with $\mu_{\bar{x}} = 66$ and $\sigma_{\bar{x}} = 14$

(b) $P(\bar{x} < 69.4) =$ 0.7788 (Round to four decimal places as needed.)

(c) $P(\bar{x} \geq 67.9) =$ 0.3339 (Round to four decimal places as needed.)

b) $P(\bar{x} < 69.4) = \text{Ncdf}(-E99, 69.4, 66, \frac{14}{\sqrt{10}}) = 0.77875104$
≈ 0.7788

c) $P(\bar{x} > 67.9) = \text{Ncdf}(67.9, E99, 66, \frac{14}{\sqrt{10}}) = 0.33390115$
≈ 0.3339

7. Suppose a geyser has a mean time between eruptions of 73 minutes. If the interval of time between the eruptions is normally distributed with standard deviation 28 minutes, answer the following questions.

(a) What is the probability that a randomly selected time interval between eruptions is longer than 87 minutes?

The probability that a randomly selected time interval is longer than 87 minutes is approximately 0.3085 (Round to four decimal places as needed.)

(b) What is the probability that a random sample of 10 time intervals between eruptions has a mean longer than 87 minutes?

The probability that the mean of a random sample of 10 time intervals is more than 87 minutes is approximately 0.0569. (Round to four decimal places as needed.)

(c) What is the probability that a random sample of 24 time intervals between eruptions has a mean longer than 87 minutes?

The probability that the mean of a random sample of 24 time intervals is more than 87 minutes is approximately 0.0072. (Round to four decimal places as needed.)

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result. Choose the correct answer below.

- ○ **A.** The probability increases because the variability in the sample mean increases as the sample size increases.
- ○ **B.** The probability increases because the variability in the sample mean decreases as the sample size increases.
- ● **C.** The probability decreases because the variability in the sample mean decreases as the sample size increases.
- ○ **D.** The probability decreases because the variability in the sample mean increases as the sample size increases.

(e) What might you conclude if a random sample of 24 time intervals between eruptions has a mean longer than 87 minutes? Choose the best answer below.

- ○ **A.** The population mean cannot be 73, since the probability is so low.
- ○ **B.** The population mean must be less than 73, since the probability is so low.
- ○ **C.** The population mean is 73 minutes, and this is an example of a typical sampling.
- ● **D.** The population mean may be greater than 73.

a) $P(X > 87) = \text{Ncdf}(87, E99, 73, 28) = 0.308537532 \approx 0.3085$

b) $P(\bar{X} > 87) = \text{Ncdf}(87, E99, 73, \frac{28}{\sqrt{10}}) = 0.0569231 \approx 0.0569$

c) $P(\bar{X} > 87) = \text{Ncdf}(87, E99, 73, 28/\sqrt{24}) = 0.00715294 \approx 0.0072$

8. The shape of the distribution of the time required to get an oil change at a 10-minute oil-change facility is unknown. However, records indicate that the mean time is 11.1 minutes, and the standard deviation is 3.5 minutes. Complete parts (a) through (c) below.

Click here to view the standard normal distribution table (page 1).[1]
Click here to view the standard normal distribution table (page 2).[2]

(a) To compute probabilities regarding the sample mean using the normal model, what size sample would be required?

Choose the required sample size below.

- ○ **A.** The normal model cannot be used if the shape of the distribution is unknown.
- ○ **B.** The sample size needs to be less than 30.
- ○ **C.** Any sample size could be used.
- ● **D.** The sample size needs to be greater than 30.

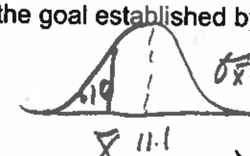
(b) What is the probability that a random sample of n = 35 oil changes results in a sample mean time less than 10 minutes?

The probability is approximately 0.0315
(Round to four decimal places as needed.)

$P(\bar{x}<10) = \text{Ncdf}(-E99, 10, 11.1, \frac{3.5}{\sqrt{35}}) = 0.031489$
≈ 0.0315

(c) Suppose the manager agrees to pay each employee a \$50 bonus if they meet a certain goal. On a typical Saturday, the oil-change facility will perform 35 oil changes between 10 A.M. and 12 P.M. Treating this as a random sample, at what mean oil-change time would there be a 10% chance of being at or below? This will be the goal established by the manager.

There would be a 10% chance of being at or below 10.3 minutes.
(Round to one decimal place as needed.)



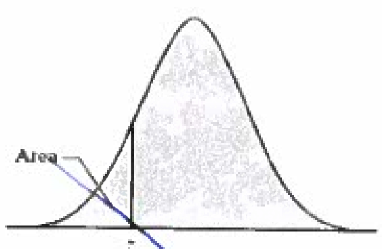
$\sigma_{\bar{x}} = \frac{3.5}{\sqrt{35}}$

1: Standard Normal Distribution Table (page 1)

$\bar{x} = \text{InvN}(.10, 11.1, 3.5/\sqrt{35}) = 10.3418 \approx 10.3$

use TI83/84

Not the tables.



Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823

1. True

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2. C. False. The sample size must be increased by a factor of four to cut the standard error in half.

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3. A.

No because the Central Limit Theorem states that regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases.

A. The sampling distribution of $\bar{x}$ is normal or approximately normal with $\mu_{\bar{x}} =$ **65** and $\sigma_{\bar{x}} =$ **0.762** .

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4. 78

4

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5. 40

2.373

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6. B. The population must be normally distributed.

B. Normal, with $\mu_{\bar{x}} = 66$ and $\sigma_{\bar{x}} = \dfrac{14}{\sqrt{10}}$

0.7788

0.3339

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7. 0.3085

0.0569

0.0072

C. The probability decreases because the variability in the sample mean decreases as the sample size increases.

D. The population mean may be greater than 73.

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8. D. The sample size needs to be greater than 30.

0.0315

10.3
