Chapter 4 Probability: The Study of Randomness

4.1. Six of the first 10 digits on line 131 correspond to "heads," so the proportion of heads is 6/10 = 60%. Although the average number of heads in 10 tosses is 5, the actual outcome is random and can vary from sample to sample.

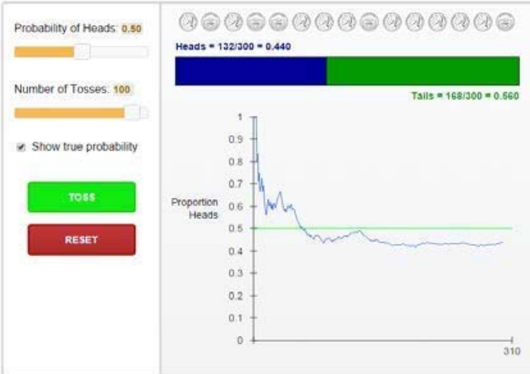
4.2. Reasons will vary. **(a)** Random (it may be "predictable" in the short term, but just saying February 2?). Weather can change dramatically from year to year. **(b)** This depends on your institution. At this author's institution, all student identification numbers begin with 900; at other institutions, the first two digits might indicate which year the student started in (e.g., 13 for 2013). **(c)** This is random.

4.3. **(a)** We can discuss the probability (chance) the temperature would be between 30 and 35 degrees Fahrenheit, for example. **(b)** Depending on your school, student identification numbers are probably not random. For example, at the author's university , all student IDs begin with 900, which means that the first three digits are all the same and not random. **(c)** The probability of an ace in a single draw is 4/52 if the deck is well-shuffled.

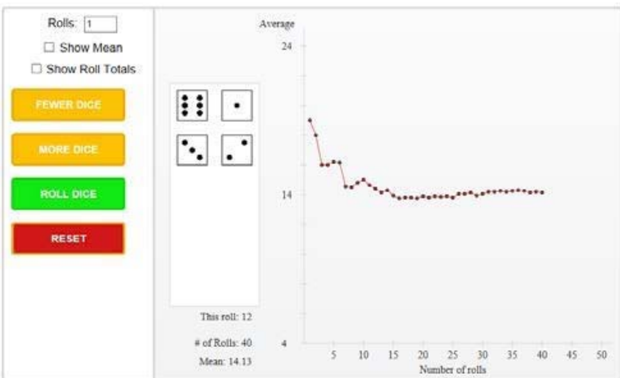
4.4. **(a)** These trials are independent. While the weather may be "cold" in February, the actual temperature on February 2 one year won't tell you much about the temperature on February 2 the next year. **(b)** These are not independent. For example, if you follow certain news outlets that tweet every Monday, you will have tweets from these same news outlets each Monday. **(c)** These are not independent; grades rely on how much you study for each course; they will depend on your aptitude and how much you study (or don' t) for each course. For example, if you study less for one course because you study more for another, your grade in the first course might suffer.

4.5. Answers will vary depending on your set of 25 rolls.

4.6. Answers will vary. In the example shown, we got 29/50 = 58% heads; the difference between the simulation and 25 heads after 50 tosses is 4 (8%). After 200 tosses, we got 86/200 = 43% heads; the difference was 14 heads (7%). **(c)** After 300 and 400 tosses, the results might look something like what is shown below. After 300 tosses, the proportion of heads was 132/300 = 0.44; after 400 tosses, the proportion of heads was 191/400 = 0.4775.



4.7. Answers will vary. This particular set of rolls had 21/40 rolls that included at least one six. Continue until you are convinced of the result.



4.8. S = {January, February, March, April, May, June, July, August, September, October, November, December}.

4.9. S = {all numbers between O and 168}.

4.10. *P(red* or brown)= P(red) + P(brown) = 0.10 +0.05 = 0.15.

4.11. A favorite color being white, black, silver, gray, or red means is it not blue, brown, or other. This probability is 1 - (0.07 + 0.05 + 0.04) = 1 - 0.16 = 0.84. Adding three probabilities and subtracting that result from 1 is slightly easier than adding the five probabilities of interest.

4.12. *P(not* a 1) = 1 - 0.301 = 0.699.

4.13. These are disjoint events, so *P(4* or (7 or more))= *P(4)* + *P(7* or more)= 0.097+0.155 = 0.252.

4.14. For each outcome (1, 2, 3, 4, 5, and 6), the probability is 1/6.

4.15. Heads and tails are equally likely on each toss, so the sample space of outcomes is {HH, HT, TH, TT}. Getting a head and then a tail is one of these, so the probability is 1/4 = 0.25.

4.16. If we know the first card was an ace, there are only three aces left in the deck and 51 cards left, so the probability the second card is an ace is 3/51. Because the outcome of the first card changes the probability for the second card (4/52 for the first and 3/51 for the second), these outcomes are not independent.

4.17. (a) S= {Yes, No}. **(b)** S= {0, 1, 2, ...,*x}*, wherex is a reasonable upper limit. **(c)** S= {18, 19, 20, ... } (presuming the friends of college students are most likely other college students). There is some leeway here with the lower and upper ends of the ages. **(d)** Answers will vary by institution. Some have many (for example, Purdue University has almost 200 possible majors); others have few (for example, a culinary school presumably has just one major).

4.18. (a) The complement rule. *P(not* A)= 1 - 0.417 = 0.583. **(b)** The addition rule for disjoint events. P(all are the same)= P(all heads)+ P(all tails)= 1/16 + 1/16 = 1/8. **(c)** The complement rule and the addition rule; at least one head and at least one tail is the complement of all heads or all tails, so 1 - (P(HHH) + P(TTT)) = 1 - 1/8 = 7/8. **(d)** No. Rule 1 says O:::::; P(a) :S 1 for any event. If *A* and Bare disjoint, the addition rule says *P(A* or *B)* = P(a) + *P(b),* which would be 0.4 + 0.8 = 1.2. This violates Rule **1. (e)** Rule 1 says O:::::; P(a) :S l for all events, so P(a) cannot be -0.04.

4.19. (a) Not equally likely: she wins close to 60% of her matches. **(b)** Equally likely. The chance of a 3 and the chance of a 4 are both 1/6. **(c)** Probably not equally likely because it depends on the intersection. **(d)** Not equally likely: home teams win more than half their games.

4.20. (a) Yes, because the two people were chosen randomly, the probability they both prefer white is (0.24)2 = 0.0576. **(b)** No, it is possible (and even likely) the two sisters have similar color preferences and therefore the events of each preferring white are not independent. **(c)** For part **(a),** the probability the first person prefers white is 0.24; this doesn't influence the probability the second person prefers white, so we again times it by 0.24. For part **(b),** the probability the first sister prefers white is 0.24, but once we know this sister's preference, it likely changes the probability that the second sister prefers white, so it is no longer 0.24.

4.21. (a) The probability that both of the two disjoint events occur is 0. (Multiplication of probabilities is appropriate for *independent* events, not *disjoint* events.) **(b)** Probabilities must be no more than l; *P(A* and *B)* will be no more than 0.7. (We cannot determine this probability exactly from the given infonnation .) **(c)** *P(notA)* = 1 -P(a) = 1- 0.45 = 0.55.

4.22. (a) The two outcomes (say, *A* and *B)* in the sample space need not be equally likely. The only requirements are that *P(*a) ::=:: 0, P(b) ::=:: 0, and *P(*a) + P(b) = 1. **(b)** In a table of random digits, each digit has probability 0.1. **(c)** If *A* and *B* were independent, then *P(A* and *B)* would equal P(a)P(b) = 0.12. In fact, the given probabilities are impossible because *P(A* and *B)* cannot be bigger than either of P(a) and P(b).

4.23. There are five possible outcomes: S = {linkl, link2, link3, link4, leave}.

4.24. There are an infinite number of possible outcomes, and the description of the sample space will depend on whether the time is measured to any degree of accuracy (Sis the set of all positive numbers) or rounded to (say) the nearest second (S = {0, 1, 2, 3, ... }), or nearest tenth of a second (S = {0, 0.1, 0.2, 0.3 ... }).

4.25. (a) P(AB) = 1 - (0.42 + 0.11 + 0.44) = 1 - 0.97 = 0.03. **(b)** *P(*can donate) = *P(0* or B) = *P(0)* + *P(b)* = 0.44 + 0.11 = 0.55.

4.26. Because they are independent, P(both 0) = (0.44)(0.52) = 0.2288. P(same type)= P(both 0) + P(both AB)+ P(both **B)** + P(both A)= (0.44)(0.52) + (0.03)(0.03) + (0.11)(0.10) + (0.42)(0.35) = 0.2288 + 0.0009 + 0.0110 + 0.1470 = 0.3877.

4.27. **(a)** No. These student categories are disjoint, and the probabilities sum to more than 1. **(b)** This is legitimate (in terms of the probability rules), but the deck would be a nonstandard one. **(c)** This is legitimate, but represents a "loaded" die (i.e., not a fair one).

4.28. **(a)** P(French) = 1 - (0.59 + 0.07 + 0.11) = 0.23. **(b)** P(not English)= 1 - 0.59 = 0.41.

4.29. **(a)** P(some education beyond high school, but no degree)= 1 -(0.12 + 0.31 + 0.29) = 0.28. **(b)** P(at least high school)= 1 - *P(* didn't finish high school)= 1 - 0.12 = 0.88.

4.30. The probabilities of 2, 3, 4, and 5 are unchanged (1/6 each), so *P(l* or 6) = 1/3. Because *P(6)* = 0.24, we must have P(l) = 1/3 - 0.24 = 0.093. The complete assignment is

Face	1	2	3	4	5	6
Probability	0.093	1/6	1/6	1/6	1/6	0.21

4.31. For example, the probability for A-positive blood is (0.35)(0.84) = 0.3528 and for A-negative blood is (0.35)(0.16) = 0.0672.

Blood type	A+	A-	B+	B-	AB+	AB-	O+	0-
Probability	0.294	0.056	0.084	0.016	0.0252	0.0048	0.4368	0.0832

4.32. **(a)** All are equally likely; the probability is 1;38. **(b)** Because 18 slots are red, the probability of a red is P(red) = 18/38 = 0.4737. **(c)** There are 12 winning slots, so P(win a column bet)= 12/38= 0.3158.

4.33. **(a)** There are six arrangements of the digits 0, 5, and 9 (059, 095, 509, 590, 905, 950), so that P(win) = 6/1000 = 0.006. **(b)** There are only three arrangements of the digits 2, 2, and 3 (223,232,322), so that *P(*win) = 3/1000 = 0.003.

4.34. **(a)** With four digits 0000 - 9999, there are 10,000 possible PINs. **(b)** The easiest way to solve this is to use the complements rule: P(at least one 0) = 1 -P(no Os)= 1 - (0.9)4 = 0.3439.

4.35. P(none are 0-negative) = P(all are not 0-negative) = $(1 - 0.07)^{10}$ = 0.4840, so P(at least one is 0-negative) = 1 -P(none are 0-negative) = 1 - 0.4840 = 0.5160.

4.36. For any event *A,* along with its complement *A* , we have *P(S)* = *P(A* or *A)* because *"A* or *A"* includes all possible outcomes (that is, it is the entire sample space S). By Rule 2, *P(S)* = **1,** and by Rule 3, *P(A* or *A)* = P(a) + *P(A),* because *A* and *A* are disjoint. Therefore, P(a) + *P(A)* = 1, from which Rule 4 follows.

4.37. Note that *A=* *(A* and *B)* or *(A* and .B"), and the events *(A* and *B)* and *(A* and .B") are disjoint, so Rule 3 says that P(a) = *P((A* and *B)* or *(A* and .B"))= *P(A* and *B)* + *P(A* and *B).* If *P(A* and *B)* = P(a)P(b), then we have *P(A* and .B") = P(a) - P(a)P(b) = P(a)(l - P(b)), which equals P(a)P(.B") by the complement rule.

4.38. The table shows the combinations. **(a)** Emily and Michael's children can only have alleles 00, so they can only have blood type 0. **(b)** P(type 0) = 1.

	0	0
O	OO	gg

4.39. The table shows the combinations. **(a)** Abdreona and Caleb's children can have alleles AA, AO, or 00, so they can have blood type A or 0. **(b)** Either note that the four combinations in the table are equally likely or compute P(type 0) = *P(O* from Nancy

	A	0
A	AA	AO
O0	AO	g

and O from David)= 0.5^2 = 0.25 and *P(type* A)= 1 - P(type 0) = 0.75.

4.40. The table shows the combinations. Samantha and Dylan's children can have alleles AB, BB, AO, or BO, so they can have blood type A, AB, or B. P(type A)= 0.25, P(type AB)= 0.25, P(type B) = 0.5. **(a)** P(both same blood type)= P(both type A)+ P(both type AB)+ P(both type B) = $0.25^2 + 0.25^2 + 0.5^2$ = 0.375 = 3/8.

	A	B
BI	AB	BBBO I
0	AO	.

4.41. The table shows the combinations. Anna and Nathan's children can have alleles AB, AO, BO or 00, so they can have any of the four blood types all with equal probability. P(type A)= P(type AB)= P(type B) = P(type 0) = 0.25. **(a)** P(at least one child has type 0) = 1 - *P(no* child has type 0) = 1 -P(all three have type A, AB, or B) = 1 - (0.75)3 = 0.578125 = 37/64. **(b)** P(all three have type 0) = 0.25^3 = 0.015625 = 1/64. P(first has type 0, next two do not)= 0.25(0. 75^2) = 0.140625 = 9/64.

	A	0
		g

4.42. Students do not satisfy the requirement with a grade of Dor F; the probability of this is 0.04 + 0.03 = 0.07.

4.43. Two tosses of a fair coin can result in RH, HT, TH, or TT. Each of these has probability 1/4. Counting the number of heads in the two tosses, we have

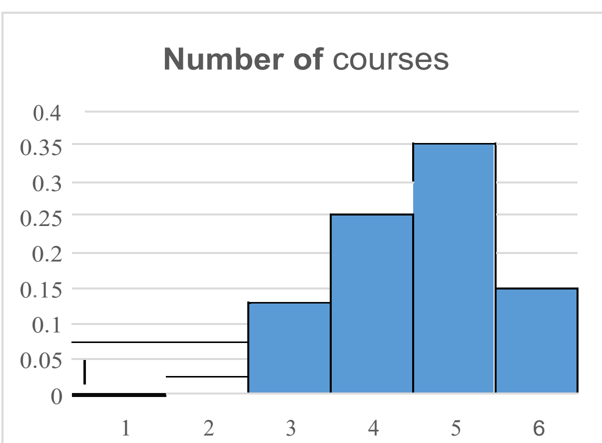
x	0	l	2
P(X =x)	0.25	0.5	0.25

4.44. *P(0.3 < X <* 0.9) = 0.9 - 0.3= 0.6.

4.45.

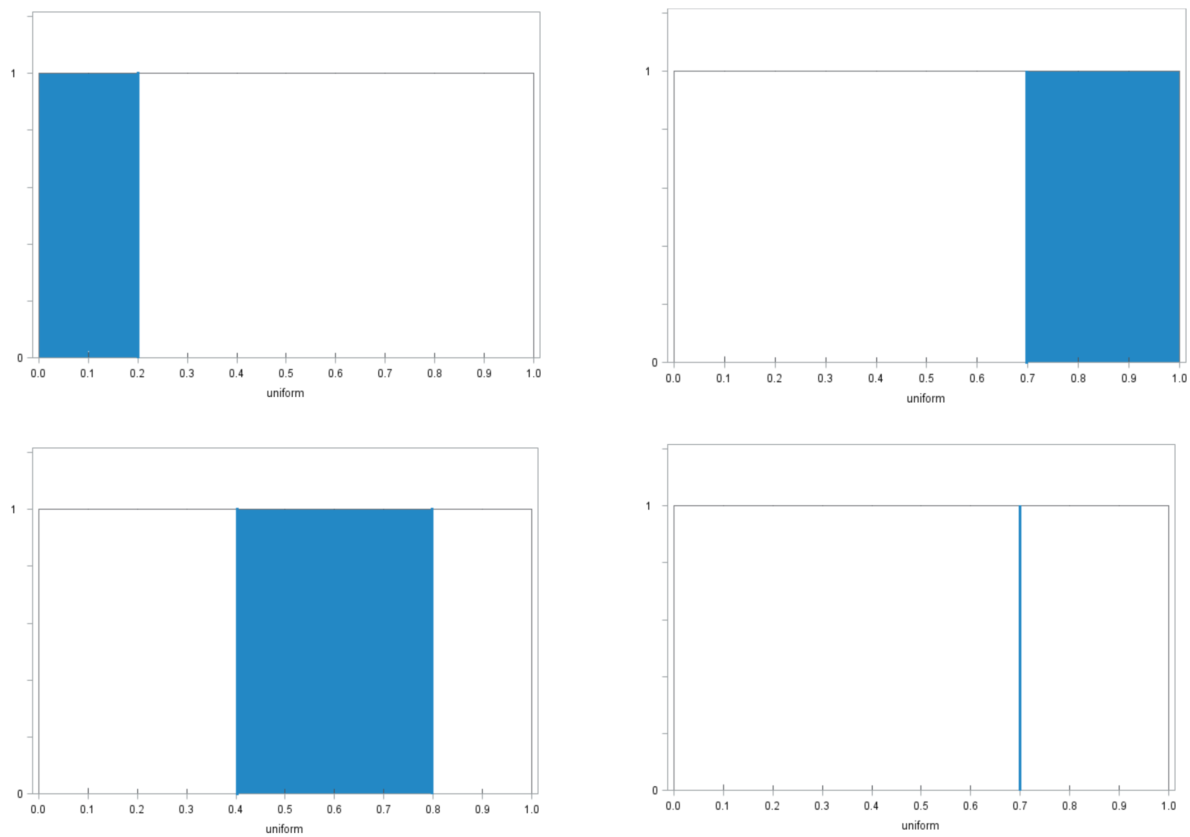
x	1	2	3	4	5	6
P(X=x)	0.05	0.05	0.13	0.26	o.36	0.15

4.46.



4.47. **(a)** *P(X"::::* 3) = *P(X=* 0) + *P(X=* l) + *P(X=* 2) = 0.05 + 0.05 + 0.13 = 0.23. **(b)** *P(X=* 4 or *X=* 5) = 0.26 + 0.36 = 0.62. **(c)** *P(X=* 8) = 0.

4.48. **(a)** *P(X <* 0.2) = 0.2. **(b)** *P(X"?.* 0.7) = 0.3. **(c)** *P(0.4 < X <* 0.8) = 0.4. **(d)** *P(X* = 0.7) = 0.



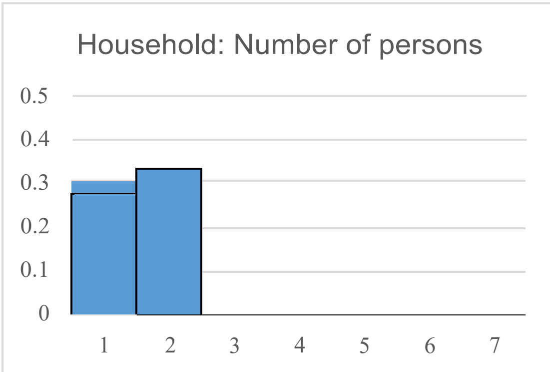
4.49. (a) The possible values certainly can be negative; it is the probabilities that can't be negative. **(b)** Continuous random variables can take values from any interval, not just Oto 1. (c) A Normal random variable is continuous. (Also, a distribution is *associated with* a random variable, but "distribution" and "random variable" are not the same things.)

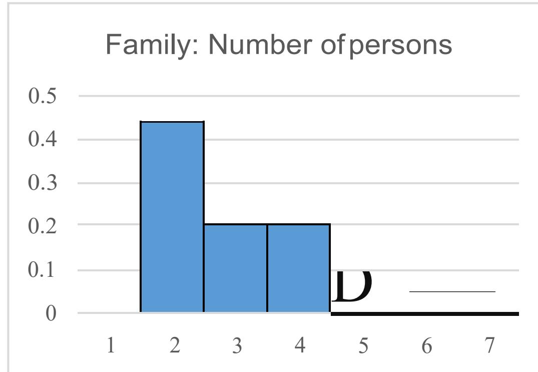
4.50. **(a)** If Tis the event that a person uses Twitter, we can write the sample space as {T, }. **(b)** There are various ways to express this: one would be S = *{TTT, TT , T T, TT, T ' T , T, 3*- **(c)** For this random variable (call itX), the sample space is {0, **1,** 2, 3}. **(d)** The sample space in part **(b)** reveals which of the three people use Twitter. This may or may not be important information; it depends on what questions we wish to ask about our sample.

4.51. Based on the information from **Exercise 4.50,** along with the complement rule, *P(T)* = 0.19 and *P(T')* = 0.81. *P(TT)* = 0.19^2 = 0.0361, *P(T)* = (0.19)(0.81) = 0.1539, *P(T)* = (0.81)(0.19) = 0.1539, and **P( )** = 0.81^2 = 06561. **(c)** Add up the probabilities from **(b)** that correspond to each value of *X*

Outcome	*TT*		*yeT*	
Probability	0.0361	0.1539	0.1539	0.6561
X	0	1	1	2
Probability	0.0361	0.3078	0.3078	0.6561

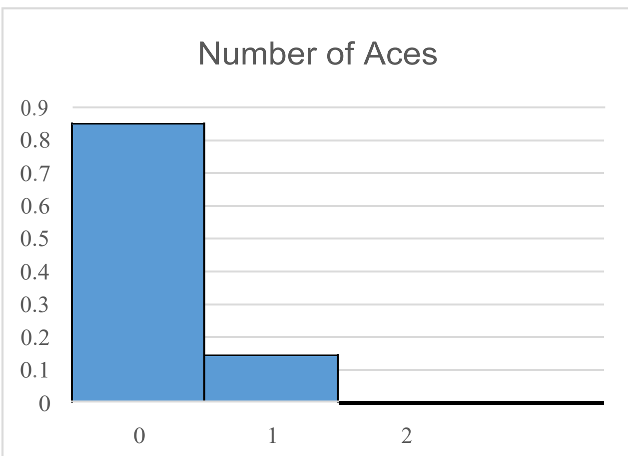
4.52. The distributions are below. *μH* = 2.52, *aH* = 1.41. *μF* = 3.11, *aF* = 1.26. The most important difference is the fact that a single person cannot be a family. This makes the family mean much larger than the household mean and also shrinks the standard deviation. Otherwise, the distribution among sizes 2-7 have similar patterns.





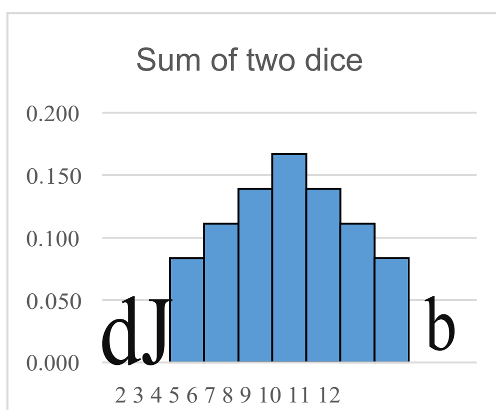
4.53. (a) Time is continuous. **(b)** Hits are discrete (you can count them). **(c)** Yearly income is discrete (you can count money).

***4.54.* (a)** 0.8507 + 0.1448 + 0.0045 = 1. It is a legitimate discrete distribution. **(b)** Shown below. **(c)** *P(X?:.* **1)** = *P(X=* **1)** + *P(X=* 2) = 0.1448 + 0.0045 = 0.1493. *P(X?:.* 1) = 1 -*P(X=* 0) = 1 - 0.8507 = 0.1493.



4.55. (a) The pairs are given below. We must assume that we can distinguish between, for example, "(1,2)" and "(2,1)"; otherwise, the outcomes are not equally likely. **(b)** Each pair has probability 1/36. **(c)** The value of Xis given below each pair. For the distribution, we see that there are four pairs that add to 5, so *P(X* = 5) = 4/36. Histogram below, right. **(d)** *P(7* or 11) = 6/36 + 2/36 = 8/36 = 2/9= 0.222. **(e)** *P(not* 7) = 1 - 6/36 = 5/6 = 0.8333.

(1, 1) 2	(1, 2) 3	(1, 3) 4	(1, 4) 5	(1, 5) 6	(1, 6) 7
(2, 1) 3	(2, 2) 4	(2, 3) 5	(2, 4) 6	(2, 5) 7	(2, 6) 8
(3, 1) 4	(3, 2) 5	(3, 3) 6	(3, 4) 7	(3, 5) 8	(3, 6) 9
(4, 1) 5	(4, 2) 6	(4, 3) 7	(4, 4) 8	(4, 5) 9	(4, 6) 10
(5, 1) 6	(5, 2) 7	(5, 3) 8	(5, 4) 9	(5, 5) 10	(5, 6) 11
(6, 1) 7	(6, 2) 8	(6, 3) 9	(6, 4) 10	(6, 5) 11	(6, 6) 12

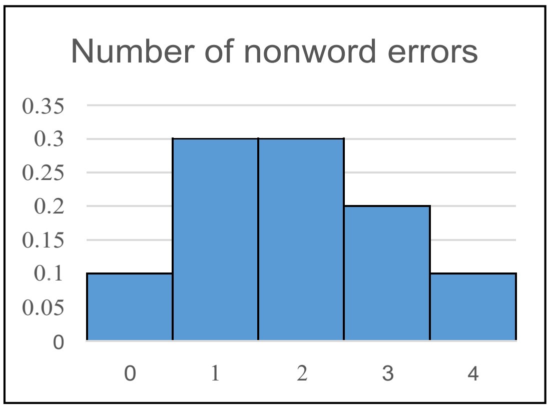


Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

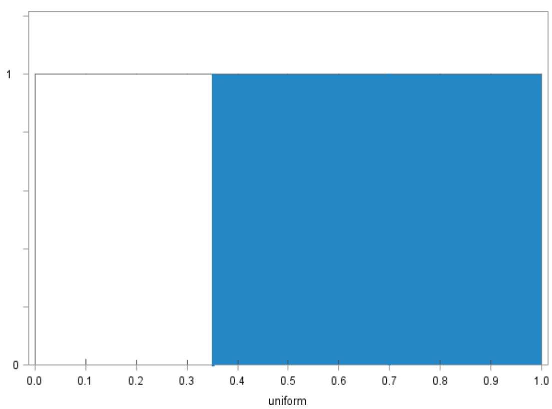
4.56. The possible values of Yare 2, 3, ... , 12, each with probability 1/12, except for 7, which has probability 2/12 or 1/6. The first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 1 (which gives a total of 2-7 with equal probability), and the rest of the time it shows 6 (which gives a total of 7-12 with equal probability).

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	3/36	3/36	3/36	3/36	3/36	6/36	3/36	3/36	3/36	3/36	3/36

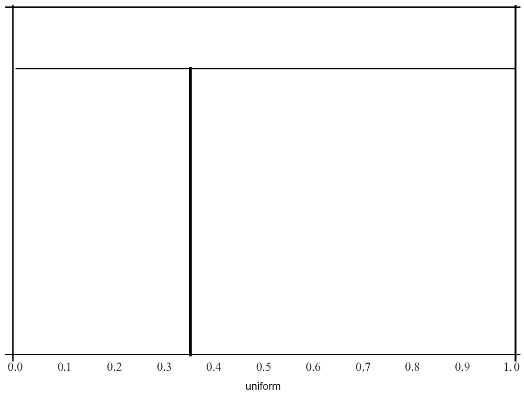
4.57. (a) Shown below. **(b)** $P(X2: 1) = 1 - P(X = 0) = 1 - 0.1 = 0.9$. **(c)** "At most, three nonword errors." $P(X\ 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.1 + 0.3 + 0.3 + 0.2 = 0.9$, $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1 + 0.3 + 0.3 = 0.7$.



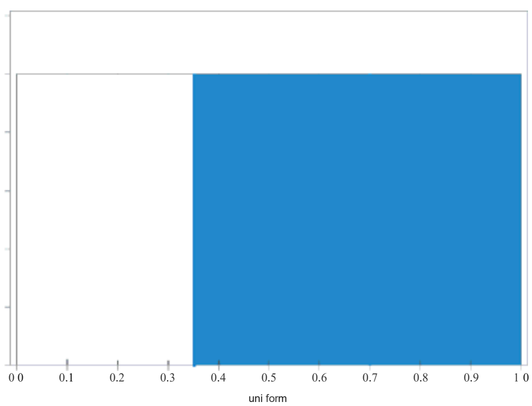
4.58. (a) $P(X2: 0.35) = 0.65$.



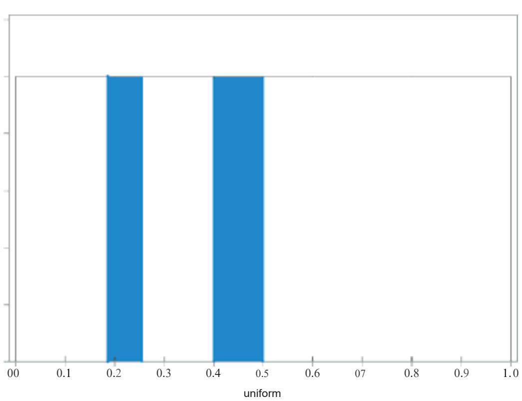
(b) $P(X = 0.35) = 0$.



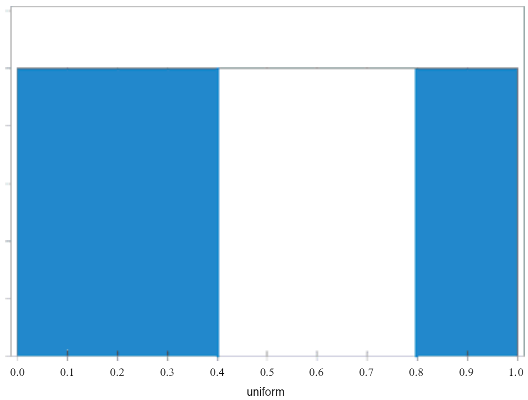
(c) $P(0.35 < X < 1.35) = P(0.35 < X < 1) = 0.65$.



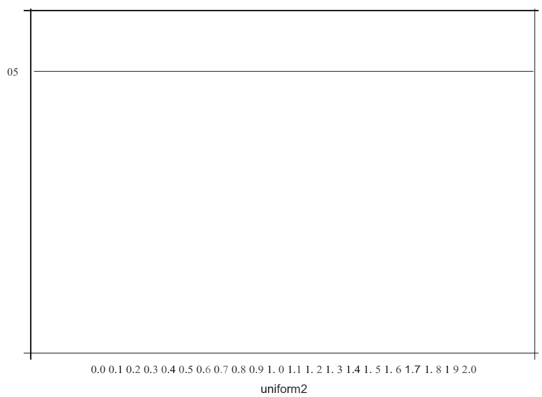
(d) P(0.18 X 0.25 or 0.4 X 0.5) = P(0.18 X 0.25) + $P(0.4$ X 0.5) = 0.07 + 0.1 = 0.17.



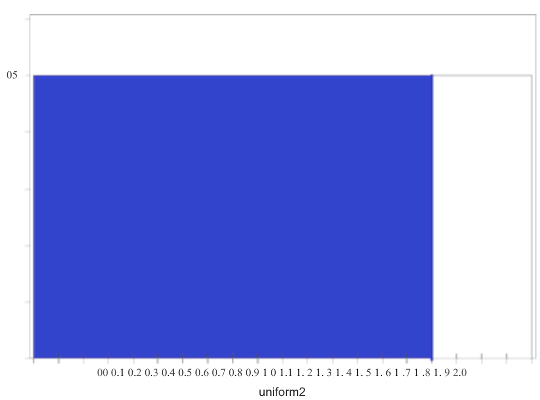
(e) 1 -*P(0.4 :::;X:::;* 0.8) = 1 - 0.4 = 0.6.



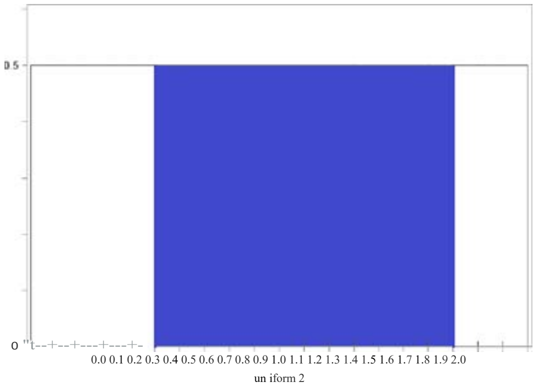
4.59. (a) The height should be 1/2 because the area under the curve must be 1.



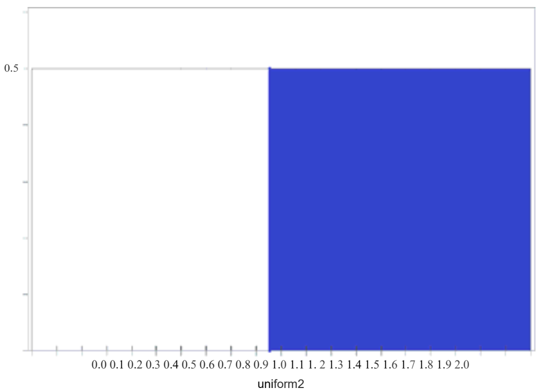
(b) $P(Y \le 1.6) = 0.8$.



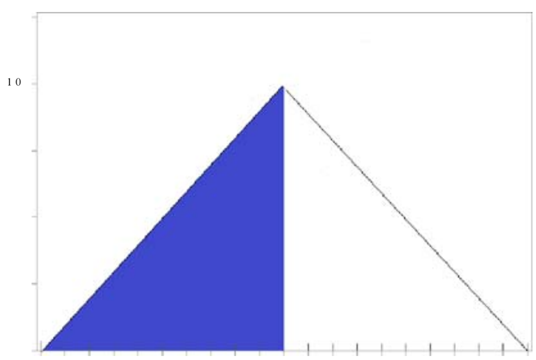
(c) $P(0.5 < Y < 1.7) = 0.6$.



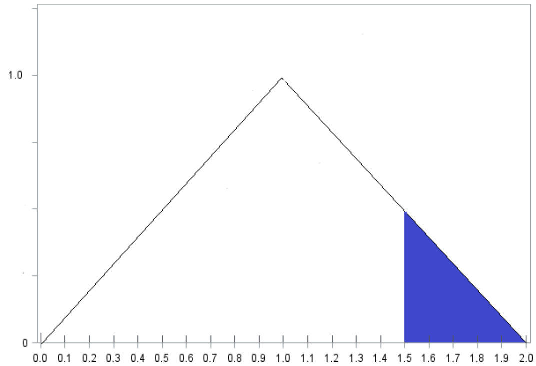
(d) *P(Y?:* 0.95) =0.525.



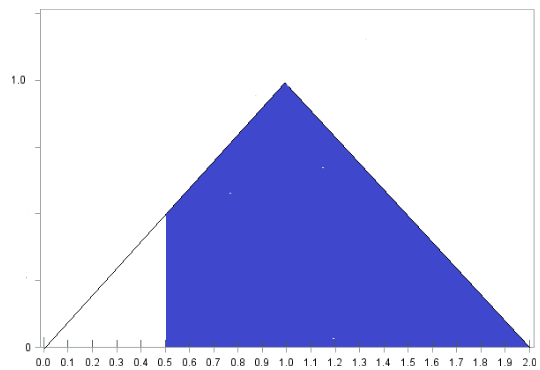
4.60. (a) Area= $0.5bh = 0.5(2)(1) = 1$. **(b)** $P(Y < 1) = 0.5$.



(c) $P(Y > 1.5) = 0.125$.



(d) $P(Y > 0.5) = 0.875$.

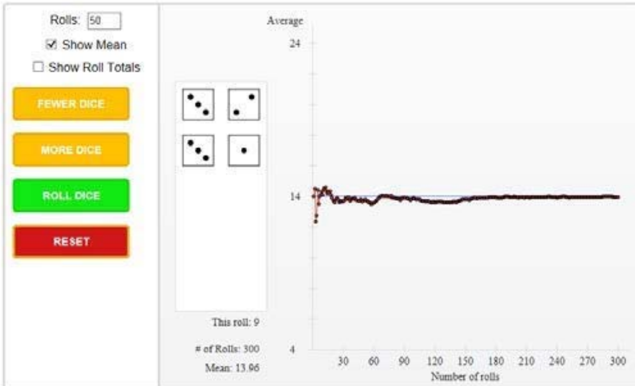


4.61. $P(8 \le \bar{X} \le 10) = P\left(\frac{8-9}{0.0724} \le \frac{\bar{X}-9}{0.0724} \le \frac{10-9}{0.0724}\right) = P(-13.8 \le Z \le 13.8)$. This probability is essentially 1; $\bar{x}$ will almost certainly estimate μ within ± 1 (in fact, it will almost certainly be much closer than this).

4.62. (a) $P(0.52 \le \hat{p} \le 0.60) = P\left(\frac{0.52-0.56}{0.019} \le \frac{\hat{p}-0.56}{0.019} \le \frac{0.60-0.56}{0.019}\right) = P(-2.11 \le Z \le 2.11) = 0.9826 - 0.0174 = 0.9652$. **(b)** $P(\hat{p} \ge 0.72) = P\left(\frac{\hat{p}-0.56}{0.019} \ge \frac{0.72-0.56}{0.019}\right) = P(Z \ge 8.42)$; this is basically 0.

4.63. The possible values of X are \$0 and \$10, each with probability 0.5. The mean amount won will be \$0(0.5) + \$10(0.5) = \$5.

4.64. Graphs will vary, but all should approach the mean as the number of "tosses" becomes large. One example is shown below; after 300 rolls, the mean sum of the four dice is 13.96, which is very close to the true mean of 14.



4.65. $\mu y = 12 + 6\mu x = 12 + 6(12) = 84$.

4.66. $\mu w = 0.5\mu u + 0.5\mu v = 0.5(25) + 0.5(25) = 25$.

4.61. $\mu x = 0(0.3) + 3(0.7) = 2.1$. $cix = (0- 2.1)2(0.3) + (3-2.1)2(0.7) = 1.89$. $ax = \quad = 1.3748$.

4.68. $\mu x = (-1)(0.2) + 0(0.3) + 1(0.2) + 2(0.3) = 0.6$.

4.69. As the sample size gets larger, the standard deviation decreases. The mean for 1000 will be much closer $to \mu$ than the mean for 2 (or 100) observations. Intuitively, as you sample more of the population, you have better information about the population, so any estimates about the population should get "better," that is, closer to the true value(s).

4.70. (a) $\mu_2 = 35 - l0\mu x = 35 - 10(30) = -265$. **(b)** $\mu_2 = 12\mu x- 5 = 12(30)- 5 = 355$. **(c)** $\mu_2 = \mu x + \mu y = 30 + 50 = 80$. **(d)** $\mu_2 = \mu x- \mu y = 30- 50 = -20$. **(e)** $\mu_2 = -2\mu x + 2\mu y = -2(30) + 2(50) = 40$.

4.71. $a^2x = (-1 - 0.6)2(0.2) + (0 - 0.6)2(0.3) + (1 - 0.6)2(0.2) + (2 - 0.6)2(0.3) = 1.24$, $a = 1.1136$.

4.72. (a) $if\ z = 10^2 a\backslash = 100(4)2 = 1600$, $CT_z = 40$. **(b)** $if\ z = l2^2{}_a{}^2{}_x = 144(4)2 = 2304$, $CT_z = 48$. **(c)** $if\ z = a\backslash + a^2 y = (4)2 + (8)2 = 80$, $CT_z = 8.944$. **(d)** $CT^2{}_z = a\backslash + a^2{}_{-Y} = (4)2 + (-1)2(8)2 = 80$, $CT_z = 8.944$. (e) $CT^2_z = a^2\text{-}2x + a^2 2y = (-2)2a^2x + (2)2a^2y = 4(4)2 + 4(8)2 = 320$, $CT_z = 17.888$.

4.73. (a) $a^2{}_2 = 10^2a^2x = 100(4)2 = 1600$, $a_2 = 40$. **(b)** $a^2{}_2 = 12^2a^2x = 144(4)2 = 2304$, $a_2 = 48$. **(c)** $a^2{}_2 = a^2x + a^2y - 2paxay = (4)2 + (8)2- 2(0.5)(4)(8) = 48$, $CT_z = 6.928$. **(d)** $a^2{}_z = a^2x + a^2{}_{-Y} = (4)2 + (-1)2(8)2- 2(0.5)(4)(8) = 48$, $CT_z = 6.928$. **(e)** $a^2{}_2 = a^2\_2x + a^2 2Y = (-2)2a\backslash + (2)2a\backslash - 2pa\_2xa2Y = (-2)2(4)2 + (2)2(8)2 - 2(0.5)(8)(16) = 192$, $CT_z = 13.856$.

4.74. (a) Each toss of the coin is independent (i.e., coins have no memory), so the probability of getting a tail on the next toss is one-half. **(b)** The variance is multiplied by $10^2 = 100$. (The mean and *standard deviation* are multiplied by 10.) **(c)** The correlation does not affect the mean of a sum (although it does affect the variance and standard deviation).

4.15. $\mu = 0(0.3) + 1(0.1) + 2(0.1) + 3(0.2) + 4(0.2) + 5(0.1) = 2.2$.

4.16. $\mu = 0(0.8507) + 1(0.1448) + 2(0.0045) = 0.1538$.

4.77. $CY = \sqrt{(0\text{-}0.1538)2(0.8507) + (1 - 0.1538)^2(0.1448) + (2\text{-}0.1538)2(0.0045)} = \sqrt{0.0201228321 + 0.1036846829 + 0.015338045} = \sqrt{0.013914556} = 0.373$.

4.78. The mean was calculated in **Exercise 4.80,** so we have (not doing any intermediate rounding)

$<J = \sqrt{(0- 2.2)2(0.3) + (1-2.2)2(0.1) + (2-2.2)2(0.1) + (3-2.2)2(0.2) + (4- 2.2)2(0.2) + (5- 2.2)2(0.1)} =$

$\sqrt{1.452 + 0.144 + 0.004 + 0.128 + 0.648 + 0.784} =$ 3J6 = 1.778.

4.79. (a) $<J = \sqrt{75^2 + 41^2}$ = \$85.48. **(b)** This is larger than that calculated in the example. The negative correlation makes the standard deviation of the sum smaller.

4.80. (a) The balance point is 1. **(b)** X_1 and X_2 each have mean 0.5 (the square will balance at its center). Indeed, A = 1 = $\mu_1 + \mu_2$.

4.81. The situation described in this exercise-"people who have high intakes of calcium in their diets are more compliant than those who have low intakes"-implies a positive correlation between calcium intake and compliance. Because of this, the variance of total calcium intake is greater than the variance we would see if there were no correlation (as the calculations in **Example 4.39** demonstrate).

4.82. If Dis the result of rolling a single four-sided die, then μn = (1 + 2 + 3 + 4)(1/4) = 2.5, and ri_D= [(1 - 2.5)2 + (2 - 2.5)2 + (3 - 2.5)2 + (4 - 2.5)2](1/4) = 1.25. Then, for the sum $I = D_1 + D_2 + l$, we have mean intelligence $\mu_1 = 2\mu_D + 1 = 6$. The variance *off* is $ci_1 = 2ci_D = 2.5$, so a_1 = I.5811.

4.83. (a) For a single toss, *X* = number of heads has possible values O and 1. μx = 0(0.5) + 1(0.5) = 0.5, and $ax = \sqrt{(0- 0.5)^2(0.5) + (0- 0.5)^2(0.5)} = \underline{0.25} = 0.5$. **(b)** Tossing four times, we have μy = 0.5 + 0.5 + 0.5 + 0.5 = 2. $<Jy = 0\sqrt{25 + 0.25 + 0.25 + 0.25} = 1 = 1$. (c) The use of the distribution is illustrated below. Summing the *xp* column, we foundμ = 2; summing the last column gives a^2 = 1, thus *a*= 1.

x	*P(X=x)*	*xp*	*(x-μ)2p*
0	0.0625	0	0.25
1	0.25	0.25	0.25
2	0.375	0.75	0
3	0.25	0.75	0.25
4	0.0625	0.25	0.25

Note: *This exercise illustrates an important fact: The result of four tosses of a coin is not the same as multiplying the result of one toss by 4. That idea works for the mean but not for the variance and standard deviation.*

4.84. Let *p* = 1, then a\ + *y* = a\+ a^2 *y + 2(1)axay* = a\+ *2axay* + a^2 *y = (ax+ ay)(ax+ ay) = (ax+ ay)2.* So, *ax* + *y* = *ax+ ay.*

4.85. (a) Because cards are dealt without replacement, the total points *X* and *Yare* not independent. **(b)** The result of one roll will not affect the other; *X* and *Y* are independent in this case.

4.86. To convert from centimeters to inches, we divide by 2.54. μin= 176.8/2.54 = 69.61 and ain = 7.2/2.54 = 2.83.

4.87. Although the probability of having to pay for a total loss for one or more of the 10 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums they collect will (almost certainly) cover the claims.

4.88. The total loss T for 10 fires has meanμ r = 10($300) = $3000, and standard deviation

a- - $\sqrt{10(\$400^{2})}$ = $400 10=$1264.91. The average loss is T/10, so μ m_0 = _!$_{\mu T}$ = $300 and amo =
$_{T}$ - IO

a- = $126.49. The total loss T for 12 fires has mean μr = 12($300) = $3600 and standard deviation *err*
10 $_{T}$

$\sqrt{12(\$400^{2})}$ = $400 12 = $1385.64. The average loss is T/12, so μ rm = _!μr= $300 and a r_{112} =
12

1
- *Or'* = $115.47.
12

Note: *The mean of the average loss is the same regardless of the number of policies, but the standard deviation decrease s as the number of policies increases. With thousands of policies, the standard deviation is very small, so the average claim will be close to $300, as was stated in the solution to the previous problem.*

4.89. P(3 or 4 or 5) = P(3) + P(4) + P(5) = 1/6 + 1/6 + 1/6 = 1/2 = 0.5.

4.90. Outcomes 2, 4, and 6 are even; the only outcome greater than 5 is 6. The event {even or greater than 5} has outcomes 2, 4, and 6. This probability is 3/6 = 1/2. Using the general addition rule, we would have 3/6 + 1/6 - 1/6 = 3/6 = 1/2.

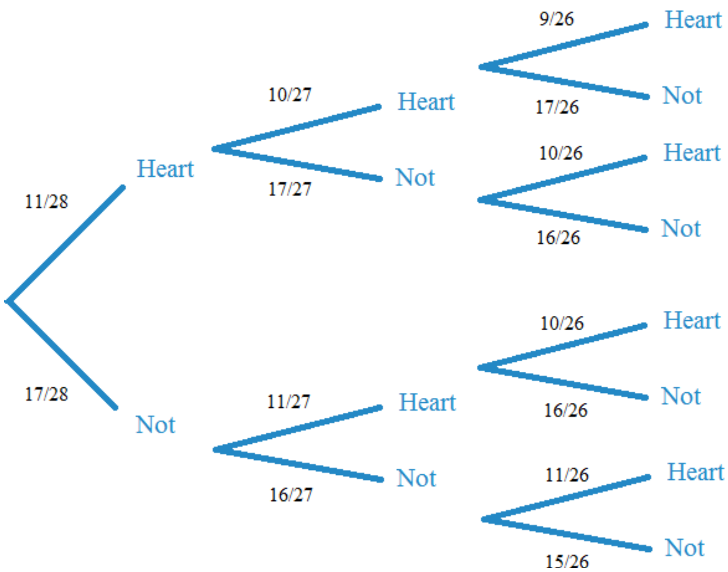
4.91. Let A be the event "next card is a heart" and B be "none of Doyle's cards are hearts." Then, $P(A \mid BJ$ = 13/48 because (other than the four known cards in Slim's hand) there are 48 cards, of which 13 are hearts.

4.92. Let A_1 = "the next card is a heart" and A_2 = "the second card is a heart." We wish to find $P(A_1$ and $A_2\}$. There are 44 unseen cards, of which 10 are hearts, so $P(A_1)$ = 10/44, and $P(A_2 \mid A_1)$ = 9/43, so $P(A_1$ and $A_2)$ = 10/44 x 9/43 = 0.0476.

4.93. This computation uses the addition rule for disjoint events, which is appropriate for this setting because B (full-time students) is made up of four disjoint groups (those in each of the four age groups).

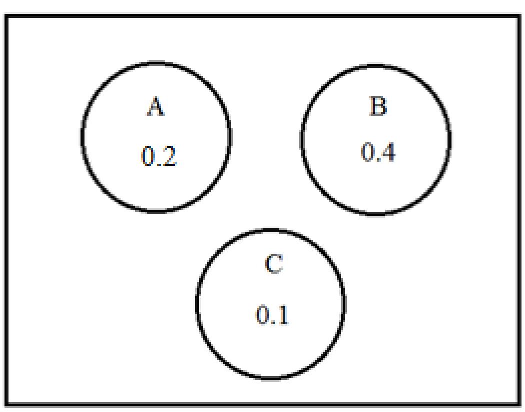
4.94. P(part-timel20 to 24 years old)= P(part-time and 20 to 24 years old)/P(20 to 24 years old)= 0.07/(0.32+0.07) =0.1795.

4.95.

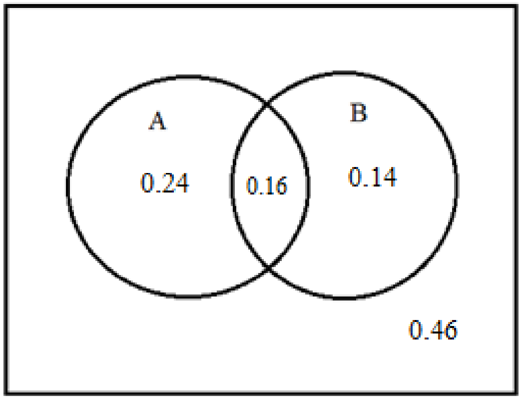


4.96. (a) No, Rule 1 says 0 :S P(a) :S **1,** so we can't have negative probabilities. **(b)** Disjoint means the events have no outcomes in common or cannot happen simultaneously. Then, *P(A* or *B)* = P(a) + *P(b)*= 0.3 + 0.5 = 0.8. **(c)** Sis the sample space. *P(S)* = 1 means that some event in the sample space must occur. **(d)** The event is called *A* complement. $P(\bar{A})$ = 1 -P(a) = 1 - 0.4 = 0.6. (e) The even *{A and* B} is the outcome that both *A* and *B* occur simultaneously. For independent events, the multiplication rule says *P(A* and *B)* = P(a)P(b) = (0.8)(0.3) = 0.24.

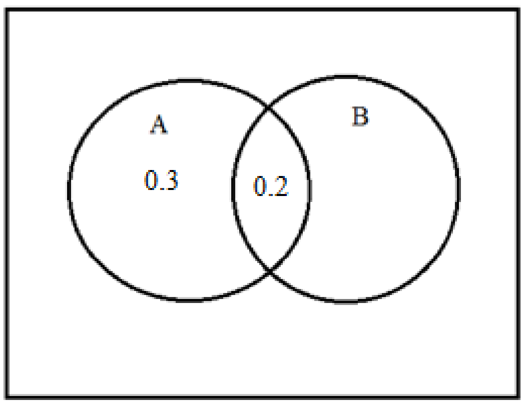
4.97. (a) For disjoint events, *P(A* or *B* or C) = P(a) + *P(b)* + *P(c)* = 0.2 + 0.4 + 0.1 = 0.7. **(b)** Shown below. **(c)** *P(A* or *B* or Cc) = 1 -*P(A* or *B* or C) = **1** -0.7 = 0.3.



***4.98.* (a)** *P(A* and *B)* = *P(a)P(BIA)* = (0.4)(0.4) = 0.16. **(b)** Shown below. **(c)** From the Venn diagram, *P(B* and $\bar{A}$) = 0.14.

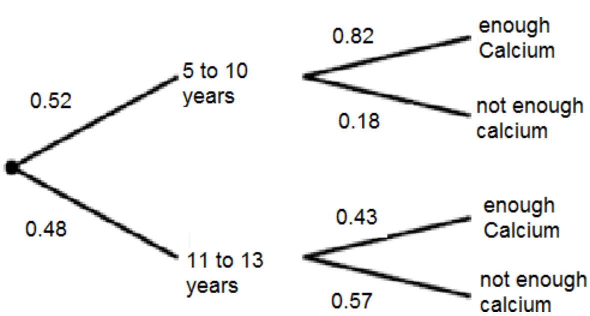


4.99. **(a)** *P(BIA) = P(A* and *B)/P(a)* = 0.2/0.5 = 0.4. **(b)** Shown below.



4.100. *P(A* and *B)* must be less than or equal to P(a). Outcomes common to both events cannot be more numerous than the outcomes in *A.*

4.101. The letters assigned here are certainly not the only possible combinations. **(a)** Let *A*= "5 to 10 years old" and *Ac*= 11 to 13 years old (these are the only two age groups under consideration). Let C = adequate calcium intake and cC= inadequate calcium intake. **(b)** P(a) = 0.52; *P(A)*= 0.48. *P(cC* ׀ *A)*= 0.18; *P(cC* ׀ *A)*= 0.57. **(c)** The tree diagram is given following. Multiply out the branches to find P(inadequate calcium) = (0.52)(0.18) + (0.48)(0.57) = 0.3672.



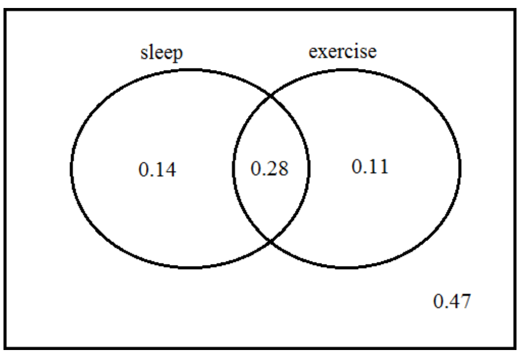
4.102. *P(l* l to 13 **׀** inadequate calcium)= $\frac{(0.4S)(0.S7)}{0.3672}$ = O.745.

4.103. These events are not independent; the probability of having inadequate calcium intake changes with the age of the child. From the solution to **Exercise 4.108,** we have *P(l* **1** to 13 |inadequate calcium) = 0.7451, which is not the same as *P(l* 1 to 13) = 0.48.

4.104. (a) This statement is true only if *A* and Bare disjoint; otherwise, *P(A* and *B)* will be added twice and can result in "probabilities" greater than 1. **(b)** The word "minus" should be "plus" because *A* n *A* = S. **(c)** The probability calculation given is for independence. Events are disjoint if *P(B* l *A)*= 0.

4.105. (a) 0.42- 0.28 = 0.14. **(b)** 0.39- 0.28 = 0.11. **(c) 1** -(0.42 + 0.39-0.28) = 0.47. **(d)** The answers in **(a)** and **(b)** are found by a variation of the addition rule for disjoint events: We note that *P(S)* = *P(S* and *E)* + *P(S* and *E*) and *P(e)* = *P(S* and *E)* + *P(SC* and *E).* In each case, we know the first two probabilities, and we find the third by subtraction. The answer for **(c)** is found by using the general addition rule to find *P(S* or *E),* and noting that SC and = (Sor *Ef*_

4.106.



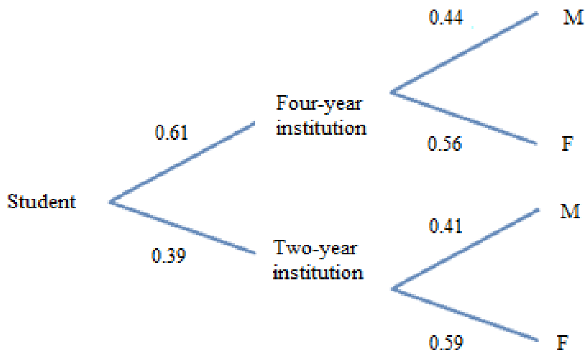
4.107. For a randomly chosen high school student, let *L* = "student admits to lying" and *M* = "student is male," so *P(L)* = 0.53, *P(M)* = 0.44, and *P(M* and *L)* = 0.24. Then, *P(M* or *L)* = *P(M)* + *P(L)* - *P(M* and *L)* = 0.44 + 0.53 - 0.24 = 0.73.

4.108. Using the addition rule for disjoint events, note that *P(if* and *L)* = *P(L)* - *P(M* and *L)* = 0.53 - 0.24 = 0.29. Then, by the defimt1on of cond1t10nal probab1hty, $P(M^{e} | L) = \frac{P(M^c \text{ and } L)}{P(L)} = \frac{0.29}{0.53} = 0.5472$.

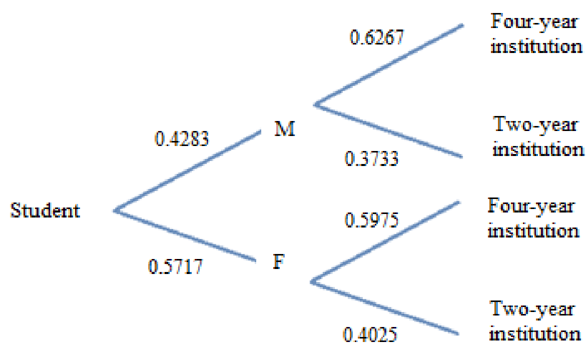
4.109. (a) Shown in the table below. **(b)** P(four-year institutionlwoman) = P(four-year institution and woman)/P(woman) = 0.3416 /(0.3416+0.2301) =0.5975.

Outcome	Men	Women
Four-year institution	0.2684	0.3416
Two-year institution	0.1599	0.2301

4.110.



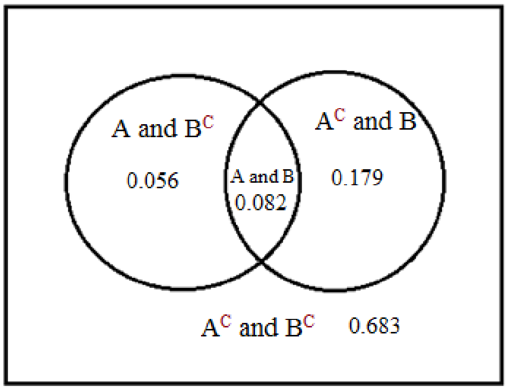
4.111. The tree diagrams are different because each branch on the tree is a conditional probability given the previous branches. So, by rearranging the order of the branches, we are necessarily changing the probabilities (assuming the events are not independent).



4.112. *P(A* or *B)* = P(a) + P(b) -*P(A* and *B)* = 0.138 + 0.261 - 0.082 = 0.317.

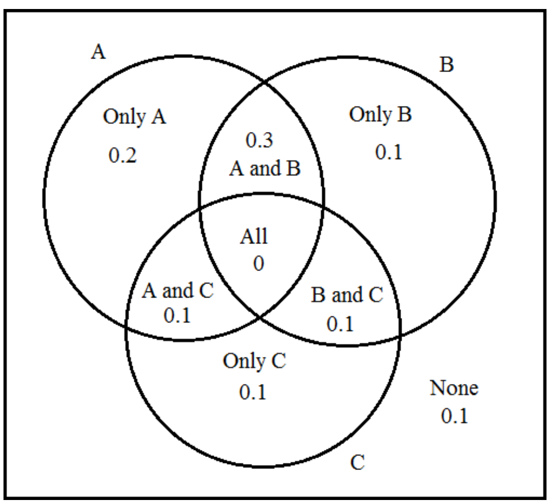
4.113. *P(AIB)* = *P(A* and *B)/P(b)* = 0.082 /0.261 = 0.3142. If *A* and Bare independent, then *P(AIB)* = P(a) but because P(a) = 0.138, *A* and Bare not independent.

4.114. (a) *(A* and *B)* is the event that the household's income exceeds $100,000 and one of the householders completed college, *P(A* and *B)* = 0.082. **(b)** *(Ac* and *B)* is the event that the household's income does not exceed $100,000 and one of the householders completed college, *P(Ac* and *B)* = 0.179. **(c)** *(A* and *Bc)* is the event that the household's income exceeds $100,000 and none of the householders completed college, *P(A* and *Bc)* = 0.056. **(d)** *(Ac* and *Bc)* is the event that the household's income does not exceed $100,000 and none of the householders completed college, *P(Ac* and *Bc)* = 0.683.



4.115. (a) The vehicle is a light truck is the event A^c, $P(A^c)$ = 0.69, given in the problem. **(b)** The vehicle is an imported car is the event *(A* and *B)*, *P(A* and *B)* = 0.08.

4.116. To find the probabilities in this Venn diagram, begin with *P(A* and Band C) = 0 in the center of the diagram. Then, each of the two-way intersections *P(A* and *B)*, *P(A* and C), and *P(B* and C) go in the remainder of the overlapping areas; if *P(A* and *B* and C) had been something other than 0, we would have subtracted this from each of the two-way intersection probabilities to find, for example, *P(A* and *B* and cC). Next, determine *P(A* only) so that the total probability of the regions that make up the event *A* is 0.6. Finally, *P(None)* = $P(A^c$ and B^c and cC) = 0.1 because the total probability inside the three sets *A*, *B*, and C is 1.



4.117. P(at least one offer)= 1 - *P(None)* = 1 - 0.1 = 0.9.

4.118. *P(A* and Band cC) = 0.3 (labeled as A and Bin the Venn diagram).

4.119. *P(B|C)* = *P(B* and C)/P (c) = 0.1/0.3 = 0.33 or 1/3. *P(C|B)* = *P(C* and *B)/P(b)* = 0.1/0.5 = 0.2 or 1/5.

4.120. **(a)** The probability is $P(A^c | B) = P(A^c$ and $B)/P(b)$ = 0.14/0.22 = 0.6364. **(b)** The two events are A^c and *B*. $P(A^c$ and $B)$ = 0.14. $P(A^c)P(b)$ = (0.69)(0.22) = 0.1518. Because these are not equal, the two events are not independent. (Note: It alternatively could be shown that $P(A^c | B)$ = 0.6364 $\neq$ 0.69 = $P(A^c)$.)

4.121. **(a)** Beth's brother has type *aa,* and he got one allele from each parent. But neither parent is albino, so neither could be type *aa.* **(b)** The table on the right shows the possible combinations, each of which is equally likely, so *P(aa)* = 0.25, *P(Aa)* = 0.5, and *P(AA)* = 0.25. **(c)** Beth is either *AA* or *Aa,* and $P(AA \mid \text{not } aa) = \frac{0.25}{0.75} = \frac{1}{3}$, while $P(Aa \mid \text{not } aa) = \frac{0.50}{0.75} = \frac{2}{3}$.

	A	*a*
A	*AA*	*Aa*
a	*Aa*	*aa*

4.122. **(a)** If Beth is *Aa,* then the first table on the right gives the (equally likely) allele combinations for a child, so *P(* child is non-albinolBeth is *Aa)* = 1/2. If Beth is *AA,* then, as the second table shows, their child will definitely be type *Aa* (and non-albino), so *P(* child is non-albinolBeth is *AA)* = **1.** **(b)** We have: *P(* child is non-albino) = *P(* child *Aa* and Beth *Aa)* + P(child *Aa* and Beth *AA)*= P(Beth *Aa)* P(child *Aa* I Beth *Aa)* + P(Beth *AA)* P(child AalBeth *AA)*= (;) (;) +(;) 1 =;, Therefore, *P(Beth* is Aalchild is *Aa)* =

	a	*a*
A	*Aa*	*Aa*
a		

	a	*a*
A	*Aa*	*Aa*
A		

$(\frac{1}{2})(\frac{3}{4}) = \pm$.

4.123. Let C be the event "Toni is a carrier," *T* be the event "Toni tests positive," and *D* be "her son has DMD." We have $P(C) = \frac{2}{3}$, P(T|C) = 0.7, and P(T|C^c) = 0.1. Therefore, *P(T)* = P(T and C) + P(T and C^c)

$= P(C)P(T|C) + P(C^c)P(T|C^c) = \frac{2}{3}(0.7) + \frac{1}{3}(0.1) = 0.5$, and $P(C|T) = \frac{P(T \cap C)}{P(T)} = \frac{(\frac{2}{3})(0.7)}{0.5} = \frac{14}{15}$ = 0.9333.

4.124. The value will be -3 approximately 30% of the time; this is because *P(-3)* = 0.3.

4.125. When repeated many times, the mean will be close to μ = 0.6(-8) + 0.4(5) = -2.4.

4.126. **(a)** μ = 2(0.2) + 3(0.8) = 2.8. σ^2 = (2 - 2.8)2(0.2) + (3 - 2.8)2(0.8) = 0.16, so σ = 0.4. **(b)** $\mu_Y = 5\mu_X - 1$ = 5(2.8) - **1** = 13. $\sigma^2_Y = 5^2\sigma^2_X$ = 4, so σ_Y = 2. **(c)** We used the rules about a linear function of a random variable and the fact that the standard deviation is the square root of the variance.

4.127. **(a)** Because the possible values of *X* are 2 and 3, the possible values of *Y* are Y = $5(2^2) - 1 = 19$ with probability 0.2 and Y = $5(3^2) - 1$ = 44 with probability 0.8. **(b)** μ = 19(0.2) + 44(0.8) = 39. σ^2 = (19. - 39)2(0.2) + (44 - 39)2(0.8) = 100, so σ = 10. **(c)** There are no rules for a quadratic function of a random variable; we must use the definitions.

4.128. **(a)** Disjoint. If the first roll is a two, it cannot be eight or higher. **(b)** Independent; the outcome of the first roll will not affect the outcome of the second. **(c)** Independent; the outcomes of the two rolls do not affect each other. **(d)** Neither. If *A* occurs, the second roll is five or less; event B is much more likely to occur.

4.129. **(a)** P(a) = 1/36, P(b) = 15/36. **(b)** P(a) = 1/36, *P(b)* = 15/36. **(c)** P(a) = 10/36, P(b) = 6/36. **(d)** P(a) = 10/36, *P(b)* = 6/36.

4.130. **(a)** μ = 2(0.4) + 3(0.3) + 4(0.3) = 2.9, σ^2 = (2 - 2.9)2(0.4) + (3 - 2.9)2(0.3) + (4 - 2.9)2(0.3) = 0.69, so σ = 0.83066. **(b - c)** Answers will vary.

4.131. For each bet, the mean is the winning probability times the winning payout, plus the losing probability times -\$10. These are summarized in the table on the right; all mean payoffs equal \$0.

Point	Expected Payoff
4 or 10	$\frac{1}{3}(+20)+\frac{2}{3}(-10)=0$
5 or 9	$\frac{2}{5}(+15)+\frac{3}{5}(-10)=0$
6 or 8	$\frac{5}{11}(+12)+\frac{6}{11}(-10)=0$

Note: *Alternatively, we can find the mean amount of money we have at the end of the bet. For example, if the point is 4 or 10, we end with either \$30 or \$0, and our expected ending amount is* $\frac{1}{3}(\$30)+\frac{2}{3}(\$0)=\$10$, *equal to the amount of the bet.*

4.132. A head on the first toss happens 50% of the time, so P(a) = 0.5. Both tosses have the same outcome happens on both heads or both tails, which is two out of four outcomes, so P(b) = 0.50.

P(BIA) = *P(B* and *A)/* P(a) = P(both heads)/P(a) = 0.25/.5 = 0.5 = P(b). Therefore, *A* and Bare independent.

4.133. (a) All the probabilities are between O and 1 and sum to 1. **(b)** P(tasters agree)= 0.03 + 0.07 + 0.25 + 0.20 + 0.06 = 0.61. **(c)** P(Taster 1 rates higher than 3) = 0.00 + 0.02 + 0.05. + 0.20 + 0.02 + 0.00 + 0.01 + 0.01 + 0.02 + 0.06 = 0.39. P(Taster 2 rates higher than 3) = 0.00 + 0.02 + 0.05 + 0.20 + 0.02 + 0.00 + 0.01 + 0.01 + 0.02 + 0.06 = 0.39.

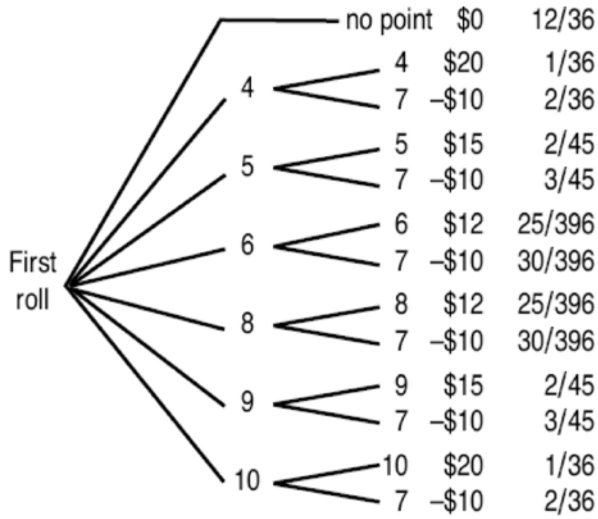
4.134. Let R1 be Taster l's rating and R_2 be Taster 2's rating. $P(R_1=3)$ = 0.01 + 0.05 + 0.25 + 0.05 + 0.01 =0.37, so: $P(R2>3 \mid R1=3)=\frac{P(R2>3 \text{ and } R1=3)}{P(R1=3)}=\frac{0.05+0.01}{0.37}=0.1622.$

4.135. This is the probability of nine (independent) losses, followed by a win; by the multiplication rule, this is $(0.994)^9(0.006) = 0.00568$.

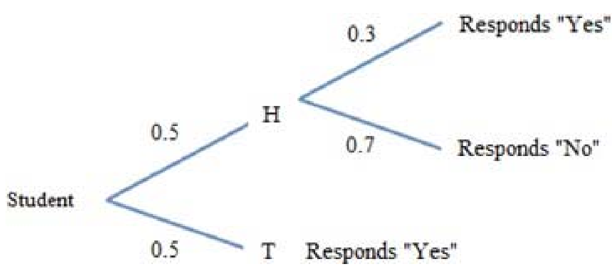
4.136. For Two-year: 1000/1721 = 58.11% are public, 721/1721 = 41.89% are private. For Four-year: 2774/3446 = 80.50% are public, 672/3446 = 19.50% are private. A much larger percent of four-year institutions are public than for two-year institutions. For Public: 1000/3774 = 26.50% are two-year, 2774/3774 = 73.50% are four-year. For Private: 721/1393 = 51.76% are two-year, 672/1393 = 48.24% are four-year. For public institutions, a much larger percent are four-year versus two-year, whereas for private institutions they are split about half and half, two-year and four-year.

4.137. *P(no* point is established)= 12/36 = 1/3. In **Exercise 4.131,** the probabilities of winning each odds bet were given as 1/3 for 4 and 10, 2/5 for 5 and 9, and 5/11 for 6 and 8. This tree diagram can get a bit large (shown below). The probability of winning an odds bet on 4 or 10 (with a net payout of \$20) is $\left(\frac{2}{36}\right)\left(\frac{!}{3}\right)=\left(\frac{!}{36}\right)$. Losing that odds bet costs \$10 and has probability $\left(\frac{2}{36}\right)\left(\frac{}{3}\right)=\frac{2}{36}$, (or $\frac{!}{18}$).

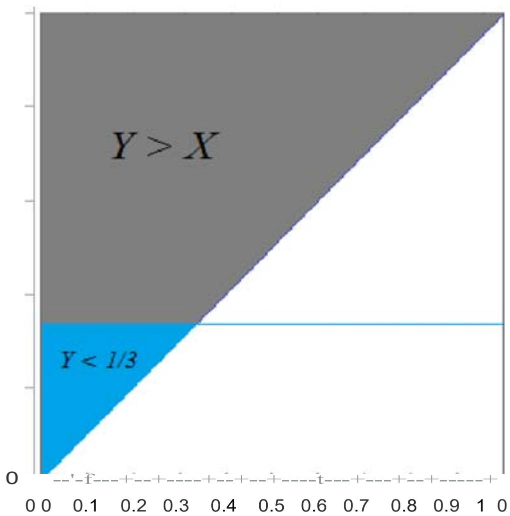
Similarly, the probability of winning an odds bet on 5 or 9 is $\left(\frac{}{36}\right)\left(\frac{}{5}\right)=\frac{2}{45}$, and the probability of losing that bet is $\left(\frac{}{36}\right)\left(\frac{}{5}\right)=\frac{2}{45}$, (or 1/15). For an odds bet on 6 or 8, we win \$12 with probability $\left(\frac{5}{36}\right)\left(\frac{5}{11}\right)=\frac{25}{396}$ and lose \$10 with $\left(\frac{5}{36}\right)\left(\frac{6}{11}\right)=\frac{30}{396}=\frac{5}{66}$. To confirm that this game 1s fair, one can multiply each payoff by its probability then add up all of those products. More directly, because each individual odds bet is fair (as was shown in the solution to **Exercise 4.131),** one can argue that taking the odds bet whenever it is available must be fair.



4.138. The probability of a "No" is 0.5(0.7) = 35%, which is half of the actual percent who have not plagiarized. If the probability of plagiarism is 0.2, the probability of a "No" would be 0.5(0.8) = 40%, again half of the actual percent who have not plagiarized. With 39% "No" answers, we estimate twice as much, 78%, to have not have plagiarized, which means we estimate that 22% of the population have indeed plagiarized a paper.



4.139. 1/18.



Y > X
Y < 1/3
0
0 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 0