

**6.6.** The confidence interval is  $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 24,164 \pm 1.96 \frac{8500}{\sqrt{1593}} = 24,164 \pm 417.41$ . The interval becomes \$23,746.59 to \$24,581.41.

**6.7.** The margin of error would be halved because  $n = 6375$  is roughly four times  $n = 1593$ . All else being equal, the width of the interval changes as  $\sqrt{n}$ .  $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 24,164 \pm 1.96 \frac{8500}{\sqrt{6375}} = 24,164 \pm 208.66$ . The confidence interval would now be half as wide as in the previous exercise because the new margin of error is roughly half as large as the old margin of error.

**6.8.** A 99% confidence interval would have a larger margin of error; a wider interval is needed in order to be more confident that the interval includes the true mean. The 99% confidence interval for  $\mu$  is  $24,164 \pm 2.576 \frac{8500}{\sqrt{1593}} = 24,164 \pm 548.60$ . This margin of error is indeed larger.

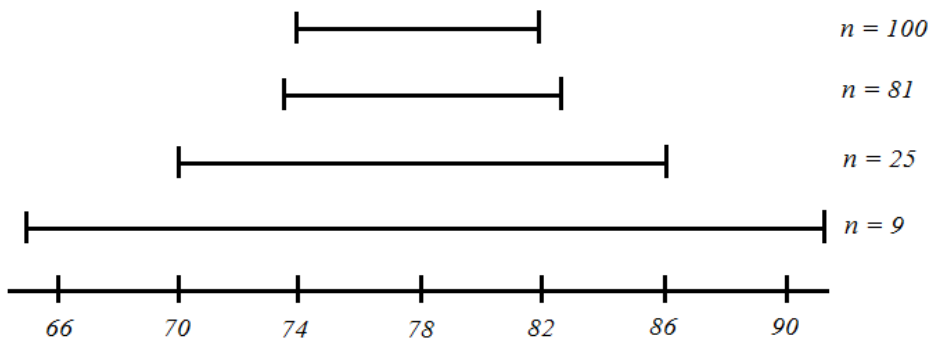
**6.9.**  $n \geq \left( \frac{1.96 * 3850}{500} \right)^2 = 227.8$ . Take  $n = 228$ . Note that you did not need the reported average salary; the margin of error for a confidence interval is based on the standard deviation, the confidence level, and the size of the sample.

**6.10.** If we need  $n = 228$  to obtain a \$500 margin of error, the margin of error for  $n = 300$  will be smaller [ $m = (1.96)(3850 / \sqrt{300}) = \$435.67$ ], if all respond. If only half respond, we will have  $n = 150$  and a larger margin of error [ $m = (1.96)(3850 / \sqrt{150}) = \$616.13$ ].

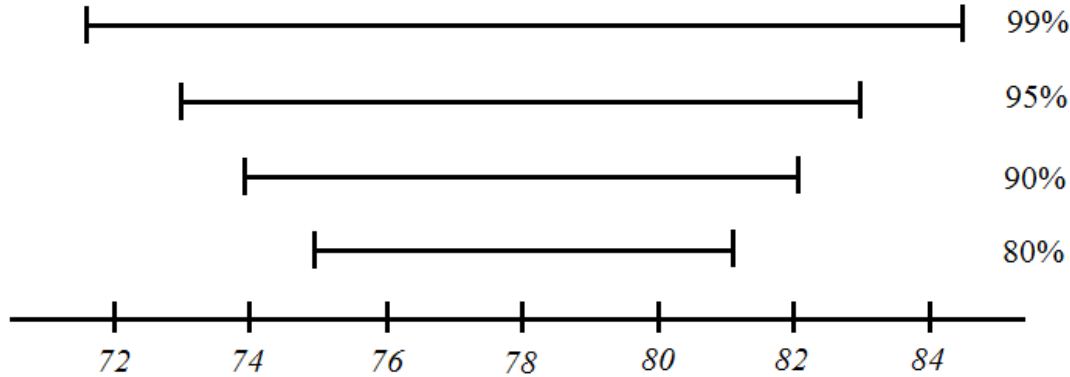
**6.11.** Answers will vary. We may not get a response for a variety of reasons. Regardless, it is likely the 532 who responded are different from those who didn't respond, so that our estimated margin of error is not a good measure of accuracy.

**6.12. (a)** The 95% confidence interval is  $78 \pm 5 = 73$ . to 83. **(b)** Greater than 5: larger confidence gives a larger margin of error.

**6.13.** The margins of error are  $(1.96)(20 / \sqrt{n})$ , which yield 13.067, 7.84, 4.356, and 3.92, respectively. (And, of course, all intervals are centered at 78.) Interval width decreases as sample size increases.



**6.14.** The margins of error are  $(z^*)(20/\sqrt{64})=4z^*$ . With  $z^*$  equal to 1.282, 1.645, 1.960, and 2.576, this yields 3.205, 4.1125, 4.9, and 6.44, respectively. (And, of course, all intervals are centered at 78.) Increasing the confidence level makes the interval wider.



**6.15. (a)** She did not divide the standard deviation by  $\sqrt{500} = 22.361$ . **(b)** Confidence intervals concern the population mean, not the sample mean. (The value of the sample mean is known to be 8.6 and is the center of the interval; it is the population mean that we do not know.) **(c)** 95% is a confidence level, not a probability. Furthermore, it does not make sense to make probability statements about the population mean  $\mu$ , once a sample has been taken and a confidence interval computed (at that point, both the interval and the unknown parameter are fixed, and the interval either contains the parameter, or it does not). **(d)** The large sample size does not affect the distribution of individual alumni ratings (the population distribution). The use of a Normal distribution is justified because the distribution of *the sample mean* is approximately Normal when the sample is large, based on the central limit theorem.

**6.16. (a)** The standard deviation should be divided by  $\sqrt{100} = 10$ , not by 100. **(b)** The correct interpretation is that (with 95% confidence) the *average* time spent at the site is between 3.23 and 4.37 hours. That is, the confidence interval is a statement about the population mean, not about the individual members; individual members would have a larger standard deviation ( $\sigma$ ) than the sample mean ( $\sigma/\sqrt{n}$ ). **(c)** To halve the margin of error, the sample size needs to be quadrupled, to about 400. In this case, the actual original margin of error was 0.57 hours (34.2 min); to get to 15 min (0.25 hours), he would need  $n = 517$  observations.

**6.17. (a)** The margin of error is  $m = 1.96 \frac{2.8}{\sqrt{720}} = 0.2045$ . The 95% confidence interval is (5.295,

5.704). **(b)** For 99% confidence, we have  $m = 2.576 \frac{2.8}{\sqrt{720}} = 0.2688$ . The 99% confidence interval is (5.231, 5.769), which is wider than the 95% confidence interval.

**6.18.** We must assume that the 720 Millennials were chosen as an SRS (or something like it). The non-Normality of the population distribution is not a problem; we have a large sample, so the central limit theorem applies.

**6.19.** For mean TRAP level, the margin of error is 2.29 U/l and the 95% confidence interval for  $\mu$  is  $13.2 \pm (1.96)(6.5/\sqrt{31}) = 13.2 \pm 2.29 = 10.91$  to 15.49 U/l.

**6.20.** For mean OC level, the 95% confidence interval for  $\mu$  is  $33.4 \pm (1.96)(19.6/\sqrt{31}) = 33.4 \pm 6.90 = 26.50$  to  $40.30$  ng/ml.

**6.21.** Scenario A has a smaller margin of error. Both samples would have the same value of  $z^*$  (1.96), but the value of  $\sigma$  would likely be smaller for A because we might expect less variability in textbook cost for freshman students than all students.

**6.22.** Subtracting \$48,127 from the confidence interval yields  $(-\$1,745, -\$119)$ . We can conclude that the average starting salary at this institution is about between \$119 and \$1745 less than the NACE mean.

**6.23. (a)** The margin of error is  $m = 1.96 \frac{540}{\sqrt{2265}} = 22.24$ . **(b)** To yield a margin of error of 15, we would need a larger sample than 2265. **(c)**  $n \geq \left( \frac{1.96 * 540}{15} \right)^2 = 4978.7$ , so we would need at least  $n = 4979$ .

**6.24. (a)**  $\sigma_{\bar{x}} = \frac{1.24}{\sqrt{175}} = 0.0937$  hours. In minutes,  $\sigma = 5.624$ . **(b)** 95% of the time, the sample mean will be between 6.5926 and 6.9674. **(c)**  $P(\bar{X} < 6.9) = P\left(Z < \frac{6.9 - 6.78}{0.0937} = 1.28\right) = 0.8997$ .

**6.25. (a)** Larger; in order to reduce the standard deviation to 5 min or 0.0833 hours, we need a larger sample. **(b)** We would need the standard deviation to be half of 5 min or 0.0833 hours, which is 0.04167.

**(c)**  $n \geq \left( \frac{1.24}{0.04167} \right)^2 = 885.5$ , so  $n = 886$ .

**6.26.** If the distribution were roughly Normal, the 68–95–99.7 rule says that 68% of all measurements should be in the range 13.8 to 53.0 ng/ml, 95% should be between  $-5.8$  and 72.6 ng/ml, and 99.7% should be between  $-25.4$  and 92.2 ng/ml. Because the measurements cannot be negative, this suggests that the distribution must be skewed to the right. The Normal confidence interval should be fairly accurate nonetheless because the central limit theorem says that the distribution of the sample mean  $\bar{x}$  will be roughly Normal.

**6.27. (a)** The 95% confidence interval for the mean number of hours spent listening to the radio in a week is  $11.5 \pm (1.96)(8.3/\sqrt{1200}) = 11.5 \pm 0.470 = 11.03$  to  $11.97$  hours. **(b)** No. This is a range of values for the mean time spent, not for individual times. (See also the comment in the solution to **Exercise 6.25.**) **(c)** The large sample size ( $n = 1200$  students surveyed) allows us to use the central limit theorem to say  $\bar{x}$  has an approximately Normal distribution.

**6.28. (a)** To change from hours to minutes, multiply by 60:  $\bar{x}_m = 60\bar{x}_h = 690$  and  $\sigma_m = 60\sigma_h = 498$  minutes. **(b)** For mean time in minutes, the 95% confidence interval for  $\mu$  is  $690 \pm (1.96)(498/\sqrt{1200}) = 690 \pm 28.18 = 661.82$  to  $718.18$  minutes. **(c)** This interval can be found by multiplying the previous interval (11.03 to 11.97 hours) by 60. This would be accurate to within rounding error.

**6.29. (a)** We can be 95% confident but not *certain*. (The true population percentage is either in the confidence interval or it isn't.) **(b)** We obtained the interval 53.1% to 55.1% by a method that gives a

correct result (that is, includes the true value of what we are trying to estimate) 95% of the time. (c) For 95% confidence, the margin of error is about two standard deviations (that is,  $z^* = 1.96$ ), so  $\sigma_{\text{estimate}} = 0.51\%$ . (d) No, confidence intervals only account for random sampling error.

**6.30. (a)** The standard deviation of the mean is  $\sigma_{\bar{x}} = 3.5/\sqrt{20} = 0.7826$  mpg. **(b)** A stemplot (not shown) does not suggest any severe deviations from Normality. The mean of the 20 sample values is  $\bar{x} = 43.17$  mpg. **(c)** If  $\mu$  is the population mean fuel efficiency, the 95% confidence interval for  $\mu$  is  $43.17 \pm 1.96 * 3.5/\sqrt{20} = 41.636$  to  $44.704$  mpg.

**6.31.** Multiply by  $\left(\frac{1.609 \text{ km}}{1 \text{ mile}}\right)\left(\frac{1 \text{ gallon}}{3.785 \text{ liters}}\right) = 0.4251 \frac{\text{kpl}}{\text{mpg}}$ . This gives  $\bar{x}_{\text{kpl}} = 0.4251\bar{x}_{\text{mpg}} = 18.3515$  and margin of error  $(1.96)(0.4251\sigma_{\text{mpg}}/\sqrt{20}) = 0.6521$  kpl, so the 95% confidence interval is 17.6994 to 19.0036 kpl.

**6.32.** Answers will vary. One sample result is shown, along with a sample stemplot of results. The number of hits will vary, but the distribution should follow a binomial distribution with  $n = 50$  and  $p = 0.95$ , so we expect the average number of hits to be about 47.5. We also find that about 99.7% of individual counts should be 43 or more, and the mean hit count for 30 samples should be approximately Normal with mean 47.5 and standard deviation 0.281, so almost all sample means should be between 46.66 and 48.34.

**6.33.**  $n \geq \left(\frac{1.96 * 6.5}{1.5}\right)^2 = 72.14$ —take  $n = 73$ .

**6.34.** If we start with a sample of size  $k$  and lose 20% of the sample, we will end with  $0.8k$ . Therefore, we need to *increase* the sample size by 25%—that is, start with a sample of size  $k = 1.25n$ —so that we end with  $(0.8)(1.25n) = n$ . With  $n = 73$ , that means we should initially sample  $k = 91.25$  (use 92) subjects.

**Note:** If a student uses the unrounded 72.14 needed subjects from the previous exercise, the calculation will yield  $(1.25)(72.14) = 90.175$ , indicating 91 will suffice

**6.35.** No, this is not trustworthy. Because the numbers are based on voluntary response rather than an SRS, the confidence interval methods of this chapter cannot be used; the interval does not apply to the whole population.

**6.36. (a)** For the mean of all repeated measurements, the 98% confidence interval for  $\mu$  is  $10.0023 \pm (2.576)(0.0002/\sqrt{6}) = 10.0023 \pm 0.00021 = 10.00209$  to  $10.00251$ g. **(b)** Because the interval in part (a) does not contain the value of 10 grams, the scale is not accurate but overestimates the weight slightly. **(c)**  $n \geq \left(\frac{(2.576)(0.0002)}{0.0001}\right)^2 = 26.54$ —take  $n = 27$ . Note this is roughly four times the sample size

**6.53. (a)** Hypotheses should be stated in terms of the population mean, not the sample mean. **(b)** The null hypothesis  $H_0$  should be that there is no change ( $\mu = 21.2$ ). **(c)** A small  $P$ -value is needed for significance;  $P = 0.98$  gives no reason to reject  $H_0$ . **(d)** We compare the  $P$ -value, not the  $z$ -statistic, to  $\alpha$ . (In this case, such a small value of  $z$  would have a very large  $P$ -value—close to 0.5 for a one-sided alternative or close to 1 for a two-sided alternative.)

**6.54. (a)** We are checking to see if the proportion  $p$  increased, so we test  $H_0: p = 0.88$  versus  $H_a: p > 0.88$ . **(b)** The professor believes that the mean  $\mu$  for the morning class will be higher, so we test  $H_0: \mu = 75$  versus  $H_a: \mu > 75$ . **(c)** Let  $\mu$  be the mean response (for the population of all students who read the newspaper). We are trying to determine if students are neutral about the change or, if they have an opinion about it, whether they think the change is an improvement, so we test  $H_0: \mu = 0$  versus  $H_a: \mu > 0$ .

**6.55. (a)** If  $\mu$  is the mean score for the population of placement-test students, then we test  $H_0: \mu = 77$  versus  $H_a: \mu \neq 77$  because we have no prior belief about whether placement-test students will do better or worse (we just wanted to know if they differ). **(b)** If  $\mu$  is the mean time to complete the maze with rap music playing, then we test  $H_0: \mu = 20$  seconds versus  $H_a: \mu > 20$  seconds because we believe rap music will make the mice finish more slowly. **(c)** If  $\mu$  is the mean area of the apartments, we test  $H_0: \mu = 880$  ft<sup>2</sup> versus  $H_a: \mu < 880$  ft<sup>2</sup> because we suspect the apartments are smaller than advertised.

**6.56. (a)** If  $p_m$  and  $p_f$  are the proportions of (respectively) males and females who like MTV best, we test  $H_0: p_m = p_f$  versus  $H_a: p_m > p_f$ . **(b)** If  $\mu_A$  and  $\mu_B$  are the mean test scores for each group, we test  $H_0: \mu_A = \mu_B$  versus  $H_a: \mu_A > \mu_B$ . **(c)** If  $\rho$  is the (population) correlation between time spent at social network sites and self-esteem, we test  $H_0: \rho = 0$  versus  $H_a: \rho < 0$ .

**Note:** In each case, the parameters identified refer to the respective populations, not the samples.

**6.57. (a)**  $H_0: \mu = \$42,800$  versus  $H_a: \mu > \$42,800$ , where  $\mu$  is the mean household income of mall shoppers. **(b)**  $H_0: \mu = 0.4$  hr versus  $H_a: \mu \neq 0.4$  hr, where  $\mu$  is this year's mean response time.

**6.58. (a)** For  $H_a: \mu > \mu_0$ , the  $P$ -value is  $P(Z > 1.89) = 0.0294$ . **(b)** For  $H_a: \mu < \mu_0$ , the  $P$ -value is  $P(Z < 1.89) = 0.9706$ . **(c)** For  $H_a: \mu \neq \mu_0$ , the  $P$ -value is  $2P(Z > 1.89) = 2(0.0294) = 0.0588$ .

**6.59. (a)** For  $H_a: \mu > \mu_0$ , the  $P$ -value is  $P(Z > -1.33) = 0.9082$ . **(b)** For  $H_a: \mu < \mu_0$ , the  $P$ -value is  $P(Z < -1.33) = 0.0918$ . **(c)** For  $H_a: \mu \neq \mu_0$ , the  $P$ -value is  $2P(Z < -1.33) = 2(0.0918) = 0.1836$ .

**6.60.** If there was, in fact, no difference in weight loss between the early and late lunch eaters, the observed difference would happen by chance only 0.2% of the time. This small chance indicates a systematic difference between early and late lunch eaters (in terms of weight loss).

**6.61.** The center of the given confidence interval is  $\bar{x} = (\$46,382 + \$48,008)/2 = \$47,195$  (the average from the sample). The margin of error for the given confidence interval is  $\$48,008 - \bar{x} = \$813$ , which means  $\sigma_{\bar{x}} = 813/1.96 = 414.8$ . Using this information, we have  $H_0: \mu = \$48,127$  versus  $H_a: \mu \neq \$48,127$ .

$z = \frac{47,195 - 48,127}{414.8} = -2.25$ .  $P$ -value =  $2P(Z < -2.25) = 2(0.0122) = 0.0244$ . We fail to reject  $H_0$ . The data does not provide evidence, at the 1% level, that the average income of graduates from your institution is different from the national average.

**6.62.**  $H_0: \mu = 369.4$  versus  $H_a: \mu \neq 369.4$ .  $z = \frac{251.2 - 369.4}{540/\sqrt{2265}} = -10.42$ .  $P$ -value =  $2P(Z < -10.42)$ , which is

essentially 0. We reject  $H_0$ . The data provides evidence, at the 5% level, that the average consumption of sweet snacks among healthy weight children has changed.

**6.63.**  $P = 0.09$  means there is some evidence for the wage decrease, but it is not significant at the  $\alpha = 0.05$  level. Specifically, the researchers observed that average wages for peer-driven students were 13% lower than average wages for ability-driven students, but (when considering overall variation in wages) such a difference might arise by chance 9% of the time, even if student motivation had no effect on wages.

**6.64.** If the presence of pig skulls were not an indication of wealth, then differences similar to those observed in this study would occur less than 1% of the time by chance.

**6.65.** Even if the two groups (the health and safety class and the statistics class) had the same level of alcohol awareness, there might be some difference in our sample due to chance. The difference observed was large enough that it would rarely arise by chance. The reason for this difference might be that health issues related to alcohol use are probably discussed in the health and safety class.

**6.66.** Even if scores had not changed over time, random fluctuation might cause the mean in 2013 to be different from the 2011 mean. However, in this case the difference for public school students was so great that it is unlikely to have occurred by chance. We therefore conclude that the public school mean increased significantly from 2011 to 2013. Private school students had a difference in means that was not large enough to be significant; thus, the difference is small enough that it likely is just attributed to chance.

**6.67.** Because the difference for public school students was statistically significant, we can say the mean score for them increased. The difference for private school students was not significant; that does not mean that it didn't increase, but rather that it didn't increase *enough* to be called significant.

**6.68.** The actual value  $P = 0.003$  tells us that the observed difference would occur no more than three times in 1000 trials by random chance. This is much more specific than saying the difference would occur no more than five times in 100 (50 in 1000) trials.

**6.69.** If  $\mu$  is the mean difference between the two groups of children, we test  $H_0: \mu = 0$  versus  $H_a: \mu \neq 0$  (we only want to know if the difference is significant). The test statistic is  $z = \frac{5.8 - 0}{1.4} = 4.14$ ,  $P$ -value = 0.00003 from software. Our data provides very strong evidence that those subjects classified as having low sleep efficiency had an average systolic blood pressure that is different than that of other adolescents.

**Note:** *The exercise reports the standard deviation of the mean difference, rather than the sample standard deviation; that is, the reported value has already been divided by  $\sqrt{238}$ .*

**6.70.** If  $\mu$  is the mean north–south location, we wish to test  $H_0: \mu = 100$  versus  $H_a: \mu \neq 100$ . We find  $z = \frac{99.74 - 100}{58 / \sqrt{584}} = -0.11$ ;  $P$ -value =  $2(0.4562) = 0.9124$ . This is not significant; we fail to reject  $H_0$ . There is not enough evidence to show that the trees are not a uniform distribution based on this test.

**6.71.** If  $\mu$  is the mean east–west location, the hypotheses are  $H_0: \mu = 100$  versus  $H_a: \mu \neq 100$  (as in the previous exercise). For testing these hypotheses, we find  $z = \frac{113.8 - 100}{58 / \sqrt{584}} = 5.75$ ,  $P$ -value  $< 0.0001$ . We reject  $H_0$ . There is strong evidence that the trees are not uniformly spread from east to west.

**6.72.**  $z = \frac{10.2 - 8.9}{2.5 / \sqrt{6}} = 1.27$ .  $P$ -value = 0.1020. Using  $\alpha = 0.05$ , we fail to reject  $H_0$ . There is not enough evidence to conclude that these sonnets were not written by our poet.

**6.73. (a)**  $z = \frac{103.3 - 95}{30 / \sqrt{25}} = 1.38$ , so the  $P$ -value =  $P(Z > 1.38) = 0.0838$ . Using  $\alpha = 0.05$ , we fail to reject  $H_0$ .

There is not enough evidence to show that the older students have a higher SSHA mean. **(b)** The important assumption is that this is an SRS from the population of older students. We also assume a Normal distribution, but this is not crucial provided there are no outliers and little skewness.

**6.74. (a)**  $H_0: \mu = 2811.5$  kcal/day versus  $H_a: \mu < 2811.5$  kcal/day. **(b)** The test statistic is  $z = \frac{2403.7 - 2811.5}{880 / \sqrt{201}} = -6.57$ , for which  $P$ -value  $< 0.0001$ . We reject  $H_0$ . There is strong evidence of below-recommended caloric consumption among female Canadian high-performance athletes.

**6.75. (a)**  $H_0: \mu = 0$  mpg versus  $H_a: \mu \neq 0$  mpg, where  $\mu$  is the mean difference. **(b)** The mean of the 20 differences is  $\bar{x} = 2.73$ , so  $z = \frac{2.73 - 0}{3 / \sqrt{20}} = 4.07$ , for which  $P$ -value  $< 0.0001$ . We reject  $H_0$ . We conclude that  $\mu \neq 0$  mpg; that is, we have strong evidence that the computer’s reported fuel efficiency differs from the driver’s computed values.

**6.76.** A sample screen (for  $\bar{x} = 1$ ) is shown below. As one can judge from the shading under the Normal curve,  $\bar{x} = 0.5$  is not significant, but 0.6 is. (In fact, the cutoff is about 0.52, which is approximately  $1.645 / \sqrt{10}$ .)