CHAPTER 4

SECTION 4.1 and 4.2 Probability Models

Experiment: Any process repeated under the same conditions does not necessarily produce the same outcome. The outcome in each trial is called experimental outcome.

Sample Space: Is the set of all possible outcomes in an experiment.

Experiment 1: Toss a coin once. $S = \{H, T\}$.

Experiment 2: Toss a coin three times.(Show the sample space through the tree diagram)

$$S = \{\text{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}\}$$

Suppose in experiments 1, and 2 we are interested on the number of heads. Let the variable X be the number of heads at any given trial. The variable X has a spread. If a variable has a spread, it must have a distribution. The distribution of the variable X, we call it a probability distribution.

Random Variable(r.v.): Is a numerical description of the experimental outcome.

Example 1: In experiment 2, let the r.v. X be the number of heads. The sample space is $S = \{0, 1, 2, 3\}$.

X=	0	1	2	3	To be a probability model or Probability distribution the following two conditions must be satisfied.
P(x)=	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1. $P(S) = \sum P(X) = 1$ 2. $P(X) \geq 0$ *for all X*

EVENT: Is the collection of experimental outcomes. The event is a subset of the sample space.

In the above example define the following events:

A: The number of heads ≥ 2. $A = \{2, 3\}$.

B: We observe exactly zero heads. $B = \{0\}$.

C: The number of heads ≤ 2. $C = \{0, 1, 2\}$.

NOTE: P(A) = P(X=2) + P(X=3) = 3/8 + 1/8 = 4/8
P(B) = P(X=0) = 1/8
P(C) = P(X=0) + P(X=1) + P(X=2) = 1/8 + 3/8 + 3/8 = 7/8

Union of two events: Given two events A and B, the union of A and B is the event containing all experimental outcomes belonging to A or B or both. The union is denoted by $A \cup B$.

Intersection: Given two events A and B, The intersection of A and B is the event containing the experimental outcomes belonging to both A and B. The intersection is denoted by $A \cap B$. If the intersection is the null set (empty set) then A and B are disjoint events.

Complement of an event is defined to be the event consisting of all sample points that are not in A. It's denoted by A^c.

$A \cup B = \{0, 2, 3\}$ $\quad$ $P(A \cup B) = \frac{1}{8} + \frac{3}{8} + \frac{1}{8} = \frac{5}{8}$

$A \cup C = \{0, 1, 2, 3\}$ $\quad$ $P(A \cup C) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1$

$B \cup C = \{0, 1, 2\}$ $\quad$ Note: $B \cup C$ is the same as the event C.

$A \cap B = \varnothing$ $\quad$ $P(A \cap B) = 0$

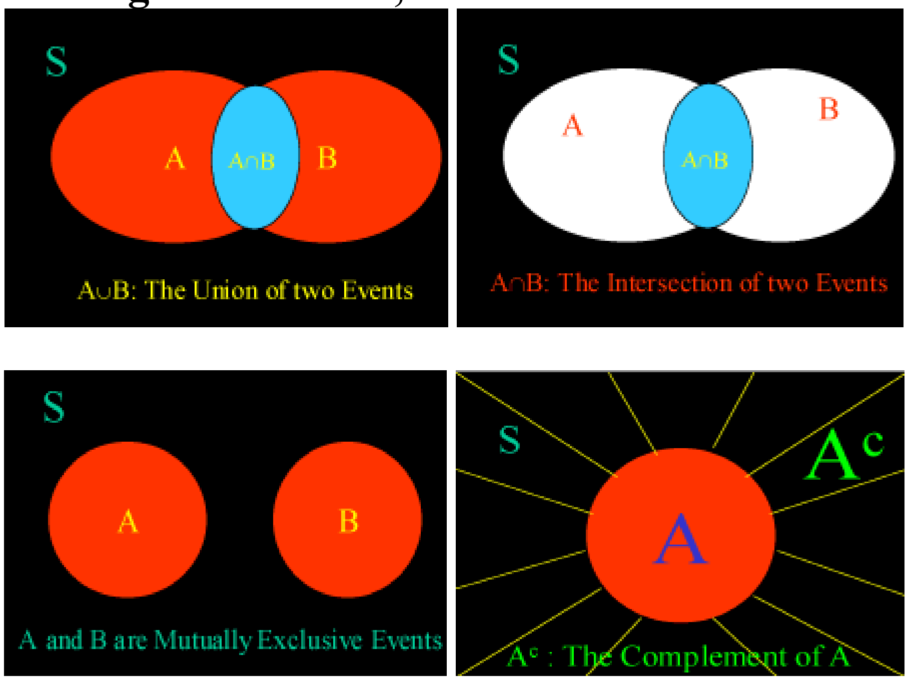
$A \cap C = \{2\}$ $\quad$ $P(A \cap C) = \frac{3}{8}$

$B \cap C = \{0\}$ $\quad$ Note: $B \cap C$ is the same as the event B.

$A=\{2, 3\}$, $A^c=\{0, 1\}$ $P(A^c)=\frac{4}{8}$ Note: $P(A^c) = 1 - P(A) = 1 - 4/8 = 4/8$

NOTE: $A \cup A^c = S$ (sample space) and $A \cap A^c = \varnothing$.

Using Venn diagrams: Union, Intersection and the Complement.



Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If the events are disjoint: $P(A \cup B) = P(A) + P(B)$.

Example 2: In a study of 100 students that had been awarded university scholarships, it was found that 40 had part-time jobs, 25 had made the dean's list the previous semester, and 15 had both a part-time job and had made the dean's list. Find the probability that a student had a part-time job or was on the dean's list?

Soln: $P(A \cup B)=P(A)+P(B)-P(A \cap B)=(40/100)+(25/100)-(15/100)=(50/100)= 0.5$

Independent Events: A and B are independent if knowing that event A occurs does not change the probability that B occurs.

Example 3: State Farm Insurance. Event A: Male driver less than 25 years old having a car accident. Event B: Male driver 25 years or older having a car accident. The events A and B are Independent.

Multiplication Rule for Independent events: $P(A \cap B) = P(A).P(B)$

In example 3, lets say that the P(A) = .20 and P(B) = .05. The insurance company would like to know the chance of both a young and an older male having an accident. $P(A \cap B) = P(A).P(B) = (.2)(.05) = .01$

Example 4: A service station manager knows from past experience that 80% of the customers use a credit card when purchasing gasoline. What is the probability that the next two customers will both pay with a credit card? Soln: 0.64

Homework: 4.21, 4.25, 4.26, 4.27, 4.28, 4.32, 4.33, 4.35, 4.38, 4.39, 4.40 pages 233-235.

Section 4.3 Random Variables

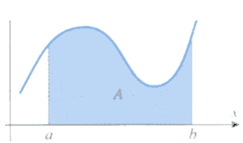
Random Variable (RV): A random variable assigns numerical value to each experimental outcome in the sample space.

Discrete Random Variable (DRV): A random variable that assumes only a finite number of values in an interval. A DRV to have a probability distribution has to satisfy the following two conditions:

$P(X_i = x_i) = P_i;\ i = 1,\ldots,k \quad where \quad 1.\ 0 \le P_i \le 1 \ for\ each\ i \quad and \quad 2.\ \sum_{i=1}^{k} P_i = 1$.

Continuous Random Variable (CRV): A random variable that assumes infinitely many number of values in an interval.

The probability is an area given under the curve. The total probability (area) is always 1 (one). $P(a \le x \le b) = P(a \prec x \prec b)$ says that the CRV X takes the values in the interval from a to b. A CRV to have a probability distribution has to satisfy the following two conditions:



1. $\int_{All\,x} f(x)\,dx = 1$ and 2. $P(a \le x \le b) \ge 0$.

Example 4: Is it discrete or continuous random variable (rv):

(a) # of defective vending machines at VSU;
(b) The amount of time required to complete your homework each day for your math2620 class;
(c) Heights of male students enrolled at VSU in Fall 99;
(d) The number of students in your math class at VSU;
(e) Distance required to stop a car traveling at 70 mph;
(f) # of chess games you will have to play before winning a game of chess.

Example 5

Probability Distribution

The instructor of a large class gives 15% each of A's and D's, 30% each of B's and C's, and 10% F's. Choose a student at random from this class. To "choose at random" means to give every student the same chance to be chosen. The student's grade on a four-point scale (with A = 4) is a random variable X.

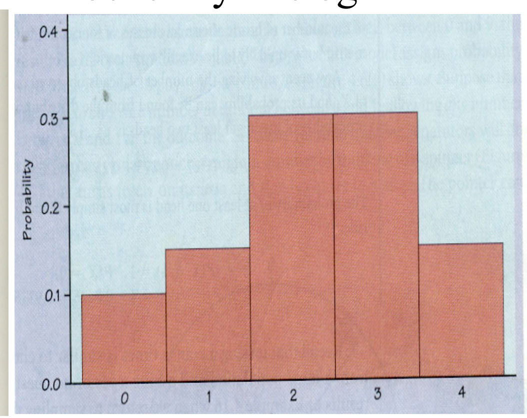
The value of X changes when we repeatedly choose students at random, but it is always one of 0, 1, 2, 3, or 4. Here is the distribution of X:

Grade	0	1	2	3	4
Probability	0.10	0.15	0.30	0.30	0.15

The probability that the student got a B or better is the sum of the probabilities of an A and a B:

$$P(\text{grade is 3 or 4}) = P(X = 3) + P(X = 4) = 0.30 + 0.15 = 0.45$$

Probability Histogram

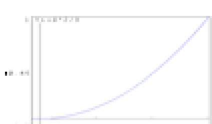


Note:The Normal RV's are denoted by $X \sim N(\mu, \sigma)$,and they are continuous.

Example 6: $Z \sim N(0, 1)$

$$P(0 \le Z \le 1) = \int_0^1 f(z)\,dz = P(Z \le 1) - P(Z \le 0) \quad (\textit{Use the tables to get the answer})$$

Example 7: A study was done on the number of years spend in prison for a theft when the sentence was 3 years. The time spend in prison is given by the following function: $f(x) = \frac{x^2}{9};\ 0 \le x \le 3$. Is this function a probability density function (pdf)?



1. *Is* $\int_{All\,x} f(x)\,dx = 1$? $\int_0^3 \frac{x^2}{9}dx = \frac{x^3}{27}\Big|_0^3 = \frac{27}{27} = 1$.

2. *Since the function is above the x − axis the* $P(a \le x \le b) \ge 0$.

1) What percentage of prisoners spends less than 2 years in prison?

$$P(X \prec 2) = \int_0^2 \frac{x^2}{9}dx = \frac{x^3}{27}\Big|_0^2 = \frac{8}{27} = 0.2963$$

2) What percentage of prisoners spends between 2 and 3 years in prison?

$P(2 \le X \le 3) = \int_2^3 \frac{x^2}{9} dx = \frac{x^3}{27}\Big|_2^3 = \frac{27}{27} - \frac{8}{27} = \frac{19}{27} = 0.7037$ or $1 - P(X \prec 2) = 1 - .2963 = 0.7037$

Uniform distribution $f(x) = \frac{1}{b-a}; \quad a \le x \le b$.

Example 8: The random number generator between 0 and 1 .

$f(x) = \frac{1}{1-0} = 1; \quad 0 \le x \le 1$. Is this function a probability density function (pdf)?

1. *Is* $\int_{All\,x} f(x)\,dx = 1$? $\int_0^1 1\,dx = x\Big|_0^1 = 1$.

2. *Since the function is above the* $x-$*axis the* $P(a \le x \le b) \ge 0$.

1) What is the probability that the number is less than 0.8?

$P(X \prec 0.8) = \int_0^{0.8} 1\,dx = x\Big|_0^{0.8} = 0.8$ Can you do it in another way?

2) What is the probability that the number is equal to 0.5?

$P(X = 0.5) = \int_{0.5}^{0.5} 1\,dx = x\Big|_{0.5}^{0.5} = 0.5 - 0.5 = 0.$

Homework: 4.48, 4.53, 4.55, 4.58, 4.59, 4.60 pages 244-245.

Section 4.4 Means and Variances of Random variables

Mean and Variance for a Discrete RV X

Expected Value of X (Mean): E(X) = μ = $\sum (xf(x))$

Variance: $\sigma_x^{\,2}$ = VAR(X) = $\sum (x-\mu)^2 f(x)$

$\sigma_x^{\,2}$ = VAR(X) = $\sum (x^2 f(x)) - \mu^2$ (computational formula)

Standard Deviation: σ_x = S.D.(X) = $\sqrt{Variance}$

Example 9: Find $E(X) = \mu$ and σ**.**

Soln.

X	f(x)=p(x)	xf(x)	x^2 f(x)
0	¼	0	0
1	2/4	2/4	2/4
2	¼	2/4	4/4

Σxf(x)= 1 6/4 =Σ x^2 f(x)

$\mu = E(X) = \sum (xf(x)) = 1$

Note: The expected value of X should be interpreted as the long run average value of X.

σ_x^2 = VAR(X) = $\sum (x^2 f(x)) - \mu^2$ = 6/4 - $(1)^2$ = 0.5

σ_x = S.D.(X) = $\sqrt{0.5}$ = 0.71

Example 10: Roll two dice. Let X = "sum of the two numbers." Find the probability function of X. Then find E(X) and S.D.(X).

Soln.

x	f(x)=p(x)	xf(x)	x^2 f(x)
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36

Σxf(x)= 252/36 1974/36 =Σx^2 f(x)

$\mu = E(X) = \sum (xf(x))$ = 252/36 = 7

σ_x^2 = VAR(X) = $\sum (x^2 f(x)) - \mu^2$ = 1974/36 – 49 = 54.83 – 49 =5.83

σ_x = S.D.(X) = $\sqrt{5.83}$ = 2.42

Mean and Variance for a Continuous RV X

Expected Value of X (Mean): E(X) = μ = $\int_{-\infty}^{\infty} x.f(x)\,dx$

Variance: $\sigma_x{}^2$ = VAR(X) = $\int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx$

$\sigma_x{}^2$ = VAR(X) = $\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$ (computational formula)

Standard Deviation: σ_x = S.D.(X) = $\sqrt{Variance}$

Example 11: Find the mean, standard deviation and median for the rv in Example 7

$f(x) = \frac{x^2}{9};\ \ 0 \le x \le 3$. $E(x) = \mu = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^3 x\frac{x^2}{9}\,dx = \int_0^3 \frac{x^3}{9}\,dx = \frac{x^4}{36}\Big|_0^3 = \frac{81}{36} = 2.25$.

The expected number of years spend in prison is 2.25 years.

Variancce: $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$\sigma^2 = \int_0^3 x^2\frac{x^2}{9}\,dx - \left(\frac{9}{4}\right)^2 = \int_0^3 \frac{x^4}{9}\,dx - \frac{81}{16} = \frac{x^5}{45}\Big|_0^3 - \frac{81}{16} = \frac{243}{45} - \frac{81}{16} = \frac{27}{5} - \frac{81}{16} = \frac{27}{80} = 0.3375\,years^2$$

Standard Deviation: $\sigma = \sqrt{\text{var}\,iance} = \sqrt{\sigma^2} = \sqrt{0.3375\,years^2} = 0.58\,years$

Median: $\int_0^M \frac{x^2}{9}\,dx = \frac{1}{2} \Rightarrow \frac{x^3}{27}\Big|_0^M = \frac{1}{2} \Rightarrow \frac{M^3}{27} = \frac{1}{2} \Rightarrow M^3 = \left(\frac{27}{2}\right)^{\frac{1}{3}} \Rightarrow M = 2.38\,years.$

Mean and Variance for a linear combination of random variables

If the rv V is a linear combination of one or more rv's , $V = ax + by$, then

$E(V) = \mu_V = aE(x) + bE(y) = a\mu_X + b\mu_y$;*That is,* $\mu_V = a\mu_X + b\mu_y$.

Variance for V is $\sigma_V^2 = a^2\sigma_X^2 + b^2\sigma_y^2$ if the rv's X and Y are INDEPENDENT.

Variance for $V = ax + by$ is $\sigma_V^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho a\sigma_x b\sigma_y$ if X and Y are Dependent.

Variance for $V = ax - by$ is $\sigma_V^2 = a^2\sigma_x^2 + b^2\sigma_y^2 - 2\rho a\sigma_x b\sigma_y$ if X and Y are Dependent.

Do examples 4.35, 4.36, 4.37, and 4.38 page 258-259.

EXAMPLE 4.35

Payoff in the Tri-State Pick 3 lottery. The payoff *X* of a $1 ticket in the Tri-State Pick 3 game is $500 with probability 1/1000 and 0 the rest of the time. Here is the combined calculation of mean and variance:

X	P(X)	X*P(X)	$X^2P(X)$
0	0.999	(0)(0.999) = 0	$(0)^2(0.999) = 0$
500	0.001	(500)(0.001) = 0.5	$(500)^2(0.001) =$ 250
		$\mu_x = \sum xp_i = 0.5$	$\sum x^2 p_i$ =250

The mean payoff is 50 cents. The variance is $\sigma_X^2 = 250 - (0.5)^2 = 249.75\ dollars^2$ with a standard deviation of $\sigma_x = \sqrt{249.5\ dollars^2} = 15.80\ dollars$ =15.80. It is usual for games of chance to have large standard deviations because large variability makes gambling exciting.

If you buy a Pick 3 ticket, your winnings are $W = X - 1$ because the dollar you paid for the ticket must be subtracted from the payoff. Let's find the mean and variance for this random variable.

EXAMPLE 4.36 Winnings in the Tri-State Pick 3 lottery. By the rules for means, the mean amount you win is $\mu_w = \mu_x - 1$ =0.5−1=−0.50 dollars.

That is, you lose an average of 50 cents on a ticket. The rules for variances remind us that the variance and standard deviation of the winnings $W = X - 1$ are the same as those of X. Subtracting a fixed number changes the mean but not the variance.

Suppose now that you buy a $1 ticket on each of two different days. The payoffs X and Y on the two tickets are independent because separate drawings are held each day. Your total payoff is $X + Y$. Let's find the mean and standard deviation for this payoff.

EXAMPLE 4.37 Two tickets. The mean for the payoff for the two tickets is $\mu_{X+Y} = \mu_X + \mu_Y$ = $0.50+$0.50=$1.00

Because X and Y are independent, the variance of $X + Y$ is $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$ =249.75+249.75=499.5. The standard deviation of the total payoff is $\sigma_{X+Y} = \sqrt{499.5}$ = $22.35.

EXAMPLE 4.38 Utility bills. Consider a household where the monthly bill for natural-gas averages $125 with a standard deviation of $75, while the monthly bill for electricity averages $174 with a standard deviation of $41. The correlation between the two bills is −0.55.

Let's compute the mean and standard deviation of the sum of the natural-gas bill and the electricity bill. We let X stand for the natural-gas bill and Y stand for the electricity bill. Then the total is $X + Y$. Using the rules for means, we have $\mu_{X+Y} = \mu_X + \mu_Y$ =125+174=299.

To find the standard deviation, we first find the variance and then take the square root to determine the standard deviation. From the general addition rule for variances of random variables,

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y = (75)^2 + (41)^2 + (2)(-0.55)(75)(41) = 3923.5$$

Therefore, the standard deviation is $\sigma_{X+Y} = \sqrt{3923.5} = 63$.
The total of the natural-gas bill and the electricity bill has mean $299 and standard deviation $63.

Homework: 4.70, 4.72, 4.73, 4.75, 4.76, 4.87, 4.88 pages 262-263.

HOMEWORK FOR A CONTINUOUS RANDOM VARIABLE

Problem 1

To travel to Atlanta, it takes anywhere between 3 to 5 hours. The distribution of time taking to drive to Atlanta is given by the following probability distribution function $f(x) = ke^{-x}$; $3 \le x \le 5$.

(a) Find k such that f(x) is a probability distribution function.
(b) Verify that f(X) is a distribution function.
(c) Find the expected time that it takes to travel to Atlanta.
(d) Find the standard deviation of the r. v. X.
(e) What is the probability that someone will do the driving in:
 (a) exactly 2 hours.
 (b) between 3 and 4 hours.
 (c) more than 4 hours.
(f) Find the median. Is the distribution symmetric or skewed? In part c you found μ_X . Where do you expect the median to be? (To the left or to the right of the mean)

Problem 2

Consider the following function: $f(x) = \frac{1}{x}$; $1 \le x \le e$.
(a) Is the function a pdf.
(b) Compute the Mean, Median and Standard Deviation.
(c) Compute the following probabilities (i) P(X > 2) (ii) P(X < 2) (iii) P(1 < X < 2).

Problem 3

A certain constant sound is described by the function $f(x) = \cos(x)$; $\pi \le x \le \frac{5\pi}{2}$. Is this function a pdf ? Show the work to support your answer.

Section 4.5 General Probability Rules

Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Do Example 4.41-(Addition Rule) on page 266

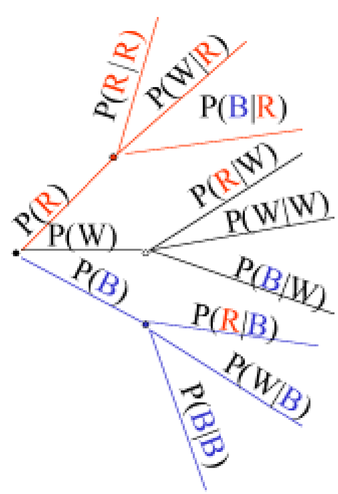
Conditional Probability: $P\langle A|B\rangle = \dfrac{P(A \cap B)}{P(B)}$ **or** $P\langle B|A\rangle = \dfrac{P(A \cap B)}{P(A)}$

Example 12: Roll a die. Let A = the number is 2, and B = the number is even. Find a) P(A) b) P(B) c) P(A $\cap$ B) d) P(A|B) Soln. a) 1/6 b) 3/6 c) 1/6 d) 1/3

Example 13: A box contains 10 balls, 3-White, 5-Blue, and 2-Red.Select 2 chips without replacement.

(a) Select 2 chips without replacement. What is the probability we observe one Blue and one White?

P(B $\cap$ W)= P(B) P(W|B) + P(W) P(B|W)= $\frac{5}{10}\cdot\frac{3}{9}+\frac{3}{10}\cdot\frac{5}{9}=\frac{30}{90}=\frac{1}{3}$.



(b) Select 3 chips without replacement. What is the probability we select Red first, White second, and Blue third?

P(R $\cap$ W $\cap$ B)= P(R) P(W|R) P(B|(R $\cap$ W))= $\frac{2}{10}\cdot\frac{3}{9}\cdot\frac{5}{8}=\frac{30}{720}=\frac{1}{24}$

Independent Events: Two events A and B are called independent if the occurrence of one does not affect the probability of the occurrence of the other. That is,

$P(A|B) = P(A)$ and $P(B|A) = P(B)$

<u>Note:</u> $P(A|B) = P(A)$ if and only if $P(B|A) = P(B)$. Thus to show that events A and B are independent, it is enough to show that $P(A|B) = P(A)$ **or** $P(B|A) = P(B)$.

Example 15: A major metropolitan police force in the eastern United States consists of 1200 officers, 960 men and 240 women. Over the past 2 years, 324 officers on the police force have been awarded promotions. The breakdown of the promotions is given in the following table. A committee of female officers charged discrimination. Are they right or wrong?

	Male	Female	Totals
Promoted	288	36	324
Not Promoted	672	204	876
Totals	960	240	1200

Solution: Get the probability table and finish it in class.

	Male	Female	Totals
Promoted	.24	.03	.27
Not Promoted	.56	.17	.73
Totals	.80	.20	1

Multiplication Rule

(a) When the events are not independent

$P(A \cap B) = P(A)P(B|A)$ or $P(A \cap B) = P(B)P(A|B)$

(b) When the events are independent

$P(A \cap B) = P(A)P(B)$

Example 16: Let A and B be events of the sample space S such that

$$P(A) = 0.7, P(B) = 0.5, P(A \cap B) = 0.35.$$

(a) Are A and B mutually exclusive?
(b) Are A and B independent?
(c) Find $P(A \cup B)$
(d) Find $P(A \cap B^c)$ (Note: $P(A \cap B^c)=P(A) - P(A \cap B)$)
(e) Find $P(A^c \cap B)$ (Note: $P(A^c \cap B)=P(B) - P(A \cap B)$)

Example 17: A service station manager knows from past experience that 80% of the customers use a credit card when purchasing gasoline. What is the probability that the next two customers will both pay with a credit card? Soln: 0.64

Example 18: I would like to purchase my dream house. My chance of getting promoted is 60%. The probability of buying the house if I get promoted is 80%. What is the probability of getting promoted and buying the house? Soln: 48%

Homework: 4.107, 4.108, 4.109, 4.112, 4.114, 4.115, 4.120 pp276-277.

Baye's Theorem
Having prior probabilities and receive new information we can always apply the Baye's Theorem to get posterior probabilities. Consider the following example:

Example 19: A manufacturing firm receives 65% of its parts from one supplier and 35% from the second supplier. The quality of the purchased parts varies with the supplier. The following table shows the percentages of good and bad parts received from the two suppliers. Let A_1 denote the event a part comes from supplier 1 and A_2 the event a part comes from supplier 2.

	Good Parts(G)	Defective Parts(D)
Supplier 1(A_1)	98%	2%
Supplier 2(A_2)	95%	5%

Given the machine broke down because of a defective part, what is the probability the defective part came from (a) supplier 1, (b) supplier 2 ?

Solution:
Given information

$$P(G \mid A_1) = 0.98 \qquad P(D \mid A_1) = 0.02 \qquad P(A_1) = 0.65$$

$$P(G \mid A_2) = 0.95 \qquad P(D \mid A_2) = 0.05 \qquad P(A_2) = 0.35$$

$$(a) \quad P(A_1 \mid D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{P(A_1 \cap D)}{P(A_1 \cap D) + P(A_2 \cap D)}$$

$$= \frac{P(A_1)P(D \mid A_1)}{P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2)} = \frac{(0.65)(0.02)}{(0.65)(0.02) + (0.35)(0.05)} = 0.426$$

$$(b) \quad P(A_2 \mid D) = \frac{P(A_2 \cap D)}{P(D)} = \frac{P(A_2 \cap D)}{P(A_1 \cap D) + P(A_2 \cap D)}$$

$$= \frac{P(A_2)P(D \mid A_2)}{P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2)} = \frac{(0.35)(0.05)}{(0.65)(0.02) + (0.35)(0.05)} = 0.574$$

Example 20: A weather satellite is sending a binary code of 0's and 1's describing a developing of a tropical storm. Channel noise, though, can be expected to introduce a certain amount of transmission error. Suppose that the message being relayed is 70% 0's and there is an 80% chance of a given 0 or 1 being received properly. If "1" is received what is the probability that a "0" was sent?

Solution:

S_0 = The event zero is send R_0 = The event zero is received

S_1 = The event one is send R_1 = The event one is received

Given information

$P(R_0 \mid S_0) = 0.8 \qquad P(R_1 \mid S_0) = 0.2 \qquad P(S_0) = 0.7$

$P(R_1 \mid S_1) = 0.8 \qquad P(R_0 \mid S_1) = 0.2 \qquad P(S_1) = 0.3$

Question: $P(S_0 \mid R_1) = ??$

$$P(S_0 \mid R_1) = \frac{P(S_0 \cap R_1)}{P(R_1)} = \frac{P(S_0 \cap R_1)}{P(S_0 \cap R_1) + P(S_1 \cap R_1)}$$

$$= \frac{P(S_0)P(R_1 \mid S_0)}{P(S_0)P(R_1 \mid S_0) + P(S_1)P(R_1 \mid S_1)} = \frac{(0.7)(0.2)}{(0.7)(0.2) + (0.3)(0.8)} = 0.3684$$

Question: $P(S_1 \mid R_0) = ??$

$$P(S_1 \mid R_0) = \frac{P(S_1 \cap R_0)}{P(R_0)} = \frac{P(S_1 \cap R_0)}{P(S_1 \cap R_0) + P(S_0 \cap R_0)}$$

$$= \frac{P(S_1)P(R_0 \mid S_1)}{P(S_1)P(R_0 \mid S_1) + P(S_0)P(R_0 \mid S_0)} = \frac{(0.3)(0.2)}{(0.3)(0.2) + (0.7)(0.8)} = 0.0968$$

Homework: Baye's Theorem

Question 1: Your next-door neighbor has a rather old and temperamental burglar alarm. If someone breaks into his house, the probability of the alarm sounding is 0.95. In the last two years, though, it has gone off on five different nights, each time for no apparent reason. Police records show that the chances of a home being burglarized in your community on any given night are 2 in 10,000. If your neighbor's alarm goes off tomorrow night, what is the probability that his house is being broken into?

Question 2: During a power blackout, 100 persons are arrested on suspicion of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty suspected and 98% reliable when given to someone who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrongdoing. What is the probability that a suspect is innocent given that the polygraph says he is guilty?