Chapter 1.1

Distributions of Random Variables

1. *Random experiment* is the experiment that has well defined outcomes and repeated under the same conditions the outcome of the experiment is uncertain.
2. *Outcome* is the result from an experiment.
3. *Sample Space* is the collection of all possible outcomes from the experiment, denoted by $\mathscr{C}$.
4. *Event* is a collection of outcomes, denoted by C. i.e. $C \subset \mathscr{C}$.

Note:
After the experiment is terminated, if the outcome is in C, then we say that the event C has occurred.
If we repeat the random experiment N times, we can count the number of times (frequency (f)) the event C has occurred.
The ratio $\frac{f}{N}$ is called the relative frequency of the event C in N times.

For small N the relative frequency tends to fluctuate up and down. As N gets larger and larger the relative frequency tends to stabilize. The number to where the relative frequency tends to stabilize we denote by P. For a large N the relative frequency of C is either P or approximately P.

Note:
We cannot predict the outcome of an experiment, but for a large N we can predict approximately the relative frequency with which the outcome of an experiment will be in C.

P has different names: The Probability of C or The Probability measure of C.

Example 1. Random experiment: Toss a fair die once.
Outcomes: 1, 2, 3, 4, 5, 6 .
Sample Space: $\mathscr{C} = \{1, 2, 3, 4, 5, 6\}$.

Example 2. Repeat the experiment in Example 1 N=600 times. Let C be the number is even. We observed the following: "1"=95, "2"=105, "3"=98, "4"=99, "5"=105, "6"=98 .

C=$\{2, 4, 6\}$. The frequency of C is $f = 105 + 99 + 98 = 302$. Since N=600, the relative frequency of C is, $\frac{f}{N} = \frac{302}{600} = 0.503\overline{3}$.

Note: In example two, the relative frequency is based on an empirical approach. Also note that it tends to stabilize to the actual Probability of C, 0.5 which it's based on the Classical approach.

Chapter 1- Homework Section 1.1

1.1. In each of the following random experiments, describe the sample space $\mathscr{C}$. Use any experience that you may have had (or use your intuition) to assign a value to the probability *P* of the event C in each of the following instances:

(a) The toss of an unbiased coin where the event C is tails.
(b) The cast of an honest die where the event C is a five or a six.
(c) The draw of a card from an ordinary deck of playing cards where the event C occurs if the card is a spade.
(d) The choice of a number on the interval zero to 1 where the event C occurs if the number is less than $\frac{1}{3}$.
(e) The choice of a point from the interior of a square with opposite vertices (-1, -1) and (1, 1) where the event C occurs if the sum of the coordinates of the point is less than $\frac{1}{2}$.

1.2. A point is to be chosen in a haphazard fashion from the interior of a fixed circle. Assign a probability *P* that the point will be inside another circle, which has a radius of one-half the first circle and which lies entirely within the first circle.

1.3. An unbiased coin is to be tossed twice. Assign a probability P_1 to the event that the first toss will be a head and that the second toss will be a tail. Assign a probability P_2 to the event that there will be one head and one tail in the two tosses.