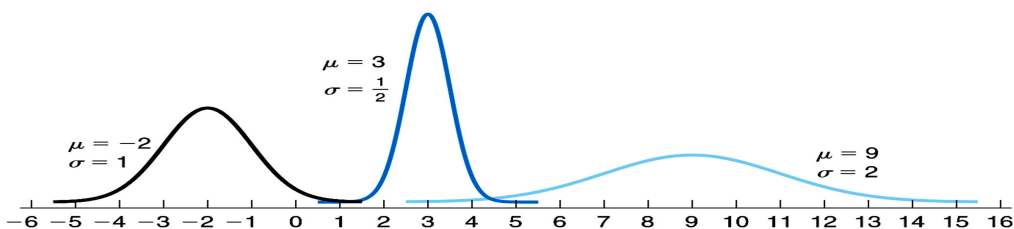


Chapter 7 -- The Normal Distribution

Sections 7.1, and 7.2 (All the work for the Normal distribution is done on the calculator. Do not use the Standard Normal tables)

In the world around us, we observe a wide variety of variables – height, weight, test scores, life of a brand of batteries, etc. Surprisingly, these variables share an important characteristic: their distributions have roughly the shape of a bell-shaped curve (normal curve) like the ones shown below.



A continuous random variable X is said to be **normally distributed** or to have a normal distribution if its distribution has the shape of a normal curve. Notation: $X \sim N(\mu, \sigma)$.

A normal distribution is completely determined by its mean and standard deviation. The probability density function of a normal random variable X is of the

form
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

Standard Normal Random Variable: A normal random variable with a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$. Such a variable is denoted by Z . That is, $Z \sim N(0, 1)$.

Use TI-83/84 to Find Area(Probability) under the Any Normal Curve

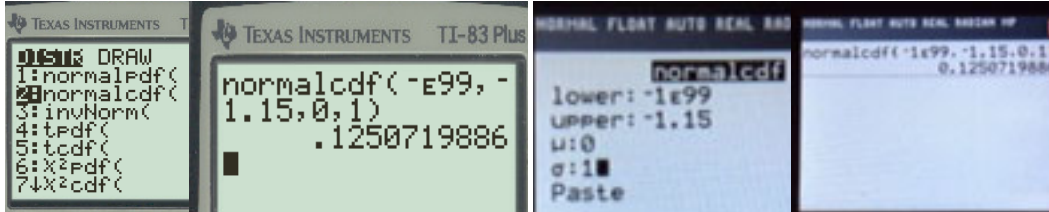
1. From the HOME screen, press 2^{nd} and VARS to access the DISTRibution menu.
2. Select 2:normalcdf(
3. Enter the Lowerbound, Upperbound, μ , and σ . Highlight Paste and hit ENTER. Hit ENTER again with the formula on the HOME screen.

Note: When there is no Lowerbound, enter -1E99. When there is no Upperbound, enter 1E99. The E shown is scientific notation; it is selected by pressing 2^{nd} then comma(,) and type 99.

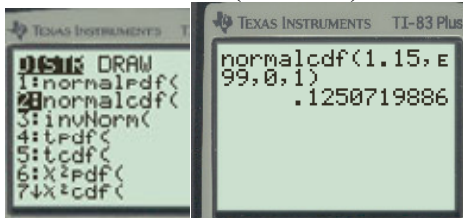
Find the probabilities using the TI 83/84 $Normalcdf$ ($\begin{matrix} Lower \\ Bound \end{matrix}$, $\begin{matrix} Upper \\ Bound \end{matrix}$, μ , σ)

Example 1:

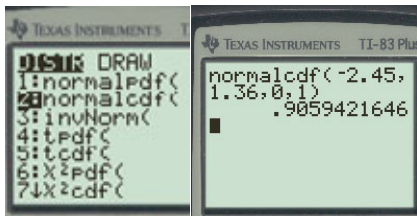
1. Determine the area under the standard normal curve that lies to the left of $Z = -1.15$; ie $P(Z \leq -1.15) = Normalcdf(-E99, -1.15, 0, 1) = 0.1251$



2. Determine the area under the standard normal curve that lies to the right of $Z = 1.15$; ie $P(Z \geq 1.15) = Normalcdf(1.15, E99, 0, 1) = 0.1251$

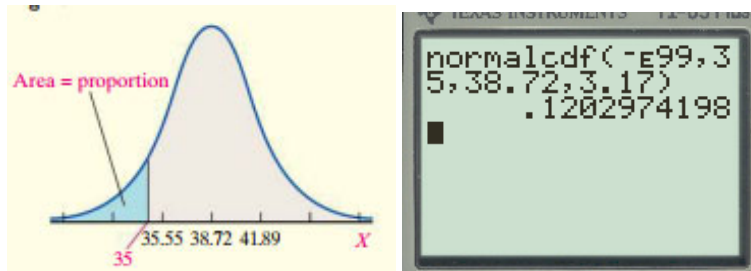


3. Determine the area under the standard normal curve that lies between $Z = -2.45$ and $Z = 1.36$; ie $P(-2.45 \leq Z \leq 1.36) = Normalcdf(-2.45, 1.36, 0, 1) = 0.9059$ or 90.59%

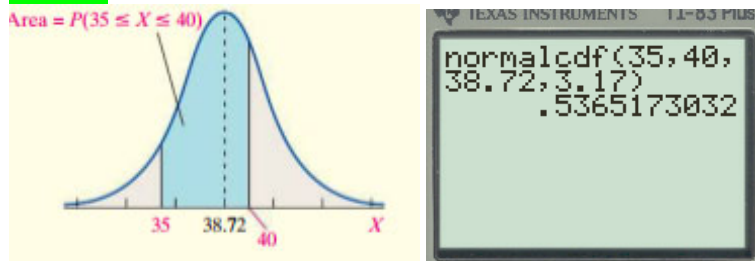


Example 2: A pediatrician obtains the heights of her three-year-old female patients. The heights are approximately normally distributed, with a mean of 38.72 inches and a standard deviation of 3.17 inches.

- (a) What is the probability that a randomly selected three-year-old female patient will have a height of less than 35, $P(X \leq 35)$ inches? 0.1203 or 12.03%



(b) What is the probability that a randomly selected three-year-old female patient will have a height between 35 and 40, $P(35 \leq X \leq 40)$ inches? 0.5365 or 53.65%



For any continuous random variable, the probability of observing a specific value of the random variable is 0. For example, for a normal random variable, $P(a) = 0$ for any value of a , because there is no area under the normal curve associated with a single value. Therefore, the following probabilities are equivalent:

$$P(a < X < b) = P(a < X \leq b) = P(a \leq X < b) = P(a \leq X \leq b) \text{ since } P(a) = 0 \text{ and } P(b) = 0.$$

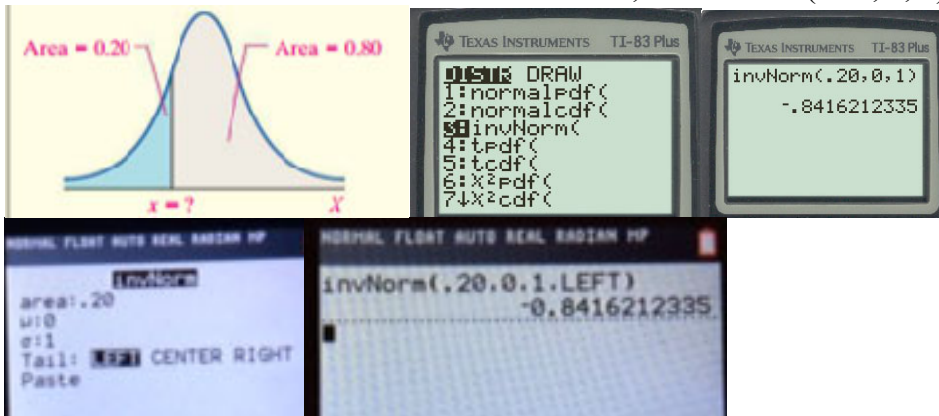
Using the TI 83/84 to Find Normal Values Corresponding to an Area

1. From the HOME screen, press 2nd and VARS to access the DISTRIBUTION menu.
2. Select 3:invNorm(.).
3. Enter the area left, μ , and σ . Highlight Paste and hit ENTER. Hit ENTER again with the formula on the HOME screen.

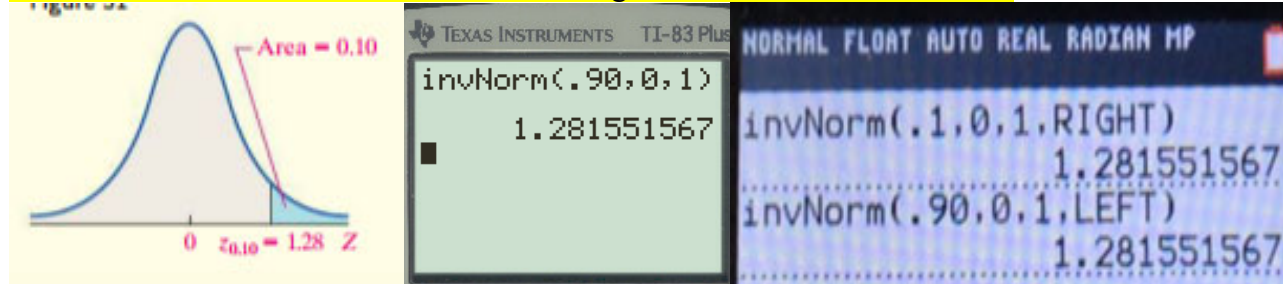
Example 3: Let $Z \sim N(0, 1)$. Find the value of Z .

The TI-83 command is: *Invnormal*(probability, μ , σ)

1. Find the Z such that the area to its left is 0.20; $\text{Invnormal}(0.20, 0, 1) = -0.8416$



2. Find the Z such that the area to its right is 0.10; $\text{Invnormal}(0.90, 0, 1) = 0.84$
 The area to the right of the unknown z-value is 0.10, so the area to the left is $1 - 0.10 = 0.90$
 For Newer calculators use 0.10 and choose “Right” or 0.90 and choose “Left”.



Note: By definition Z_α means, find the value of Z that has an area to its right α

Find the value of Z_α if $\alpha = 0.05$; i.e. Find $Z_{0.05}$; $\text{Invnormal}(0.95, 0, 1) = 1.6448$

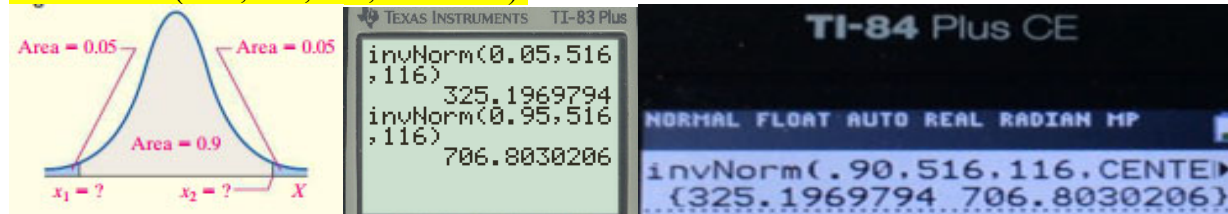
The area to the right of the unknown z-value is 0.05, so the area to the left is $1 - 0.05 = 0.95$

Find the value of $Z_{\frac{\alpha}{2}}$ if $\alpha = 0.20$; i.e. Find $Z_{0.10}$; $\text{Invnormal}(0.90, 0, 1) = 1.2815$

Example 4: The scores earned on the mathematics portion of the SAT, a college entrance exam, are approximately normally distributed with a mean of 516 and a standard deviation of 116. What scores separate the middle 90% of test takers from the bottom and the top 5%? In other words, find the 5th and 95th percentiles.

Note: With older calculators we have to do two, i.e. $\text{Invnormal}(0.05, 516, 116) = 325.2 \approx 325$ and $\text{Invnormal}(0.95, 516, 116) = 706.8 \approx 707$

Note: With newer calculators, we do only one with the middle 0.90 and choose the “CENTER”, i.e. $\text{invnorml}(0.90, 516, 116, \text{CENTER})$.



Interpretation: SAT mathematics scores that separate the middle 90% of the scores from the bottom and top 5% are 325 and 707. Put another way, a student who scores 325 on the SAT math exam is at the 5th percentile. A student who scores 707 on the SAT math exam is at the 95th percentile. We might use these results to identify those scores that are unusual.

Example 5: The personnel manager of a large company requires job applicants to take a certain test and achieve a score of at least 500. If the test scores are normally distributed with a mean of 485 and a standard deviation of 30, what percentage of the applicants pass the test? What is the 90th percentile? (a) $\text{normalcdf}(500, E99, 485, 30)=0.3085$ (b) $\text{invnormal}(.90, 485, 30)= 523.45$

Example 6: Experience indicates that the development time for a photographic printing paper is normally distributed with a mean of 30 seconds and a standard deviation of 1.1 seconds. Find the probability that it will take between 28.5 and 31.2 seconds for a randomly selected piece of photographic printing paper to develop. $\text{normalcdf}(28.5, 31.2, 30, 1.1)= 0.7752$

Example 7: The manufacturing process of ball bearings is Normally distributed with a mean diameter of 3 mm and a standard deviation of 0.1 mm. A customer has a specification that requires that ball bearings have a diameter between 2.85 and 3.1 mm. (a) What fraction of ball bearings manufactured meet specifications? (b) What fraction of ball bearings manufactured do not meet specifications? (a) $\text{normalcdf}(2.85, 3.1, 3, 0.1) = 0.7745$, (b) $1 - 0.7745 = 0.2255$

Example 8: The test scores in Math 2620 are Normally distributed with a mean of 60 and a standard deviation of 8. (a) What is the probability that a selected student has a score less than 73? (b) More than 56? (c) and (d) The instructor would like to assign F's to the lowest 15% of the class and A's to the top 5% of the class. What is the cut-off point?

(a) $\text{normalcdf}(-E99, 73, 60, 8) = 0.9479$,

(b) $\text{normalcdf}(56, E99, 60, 8) = 0.6915$,

(c) $\text{invnormal}(.15, 60, 8) = 51.7$, the cut-off point for an F is 51.7

(d) $\text{invnormal}(1-.05, 60, 8) = 73.16$, the cut-off point for an A is 73.16.

Homework-Section 7.1 and 7.2 Online - MyStatLab.