Student: ____________ **Date:** ____________	**Instructor:** Andreas Lazari **Course:** Math1111-Summer2018	**Assignment:** Section 4.5 Homework

1. The exponential model $A = 361.8\,e^{0.007t}$ describes the population, A, of a country in millions, t years after 2003. Use the model to determine the population of the country in 2003.

$A = 361.8\,e^{0.007(0)} = 361.8$

The population of the country in 2003 was 361.8 million.

2. The exponential model $A = 316.1\,e^{0.028t}$ describes the population, A, of a country in millions, t years after 2003. Use the model to determine when the population of the country will be 430 million.

$430 = 316.1\,e^{.028t} \Rightarrow \frac{430}{316.1} = e^{.028t}$

$\Rightarrow \ln\left(\frac{430}{316.1}\right) = .028t \Rightarrow t = \frac{\ln\left(\frac{430}{316.1}\right)}{.028} = 10.9 \approx 11$

The population of the country will be 430 million in 2014.
(Round to the nearest year as needed.) 11 years

3. An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16\,e^{-0.000121t}$ describes the amount of carbon-14 present after t years. Use the model to determine how many grams of carbon-14 will be present in 8940 years.

The amount of carbon-14 present in 8940 years will be approximately 5 grams. $A = 16\,e^{-.000121(8940)}$
(Round to the nearest whole number.) $= 5.42408 \approx 5$

4. The half-life of the radioactive element unobtanium-47 is 5 seconds. If 48 grams of unobtanium-47 are initially present, how many grams are present after 5 seconds? 10 seconds? 15 seconds? 20 seconds? 25 seconds?

The amount left after 5 seconds is 24 grams. In 5 sec $\frac{1}{2}(48) = 24$

The amount left after 10 seconds is 12 grams. Another 5 sec $= \frac{1}{2}(24)$ 12.

The amount left after 15 seconds is 6 grams. Another 5 sec $= \frac{1}{2}(12) = 6$

The amount left after 20 seconds is 3 grams. Another 5 sec $= \frac{1}{2}(6) = 3$

The amount left after 25 seconds is 1.5 grams. Another 5 sec $= \frac{1}{2}(3) = 1.5$
(Round to one decimal place.)

5. Prehistoric cave paintings were discovered in a cave in France. The paint contained 26% of the original carbon-14. Use the exponential decay model for carbon-14, $A = A_0\,e^{-0.000121t}$, to estimate the age of the paintings.

$.26A_0 = A_0\,e^{-.000121t} \Rightarrow .26 = e^{-.000121t}$

The paintings are approximately 11133 years old. (Round to the nearest integer.) $\Rightarrow \ln(.26) = -.000121t$

$\Rightarrow t = \ln(.26)/-.000121 = 11132.84 \approx 1113$

6. Complete the table shown to the right for the half-life of a certain radioactive substance.

Half-Life	Decay Rate, k
	7.3% per year = − 0.073

$\frac{1}{2}A_0 = A_0\,e^{-.073t} \Rightarrow \ln(.5) = \ln e^{-.073t}$

The half-life is 9.5 years.
(Round to one decimal place as needed.)

$\Rightarrow \ln(.5) = -.073t \Rightarrow t = \frac{\ln(.5)}{-.073} = 9.495 \approx 9.5$

7. Complete the table shown to the right for the half-life of a certain ~~radioactive substance~~.

Half-Life	Decay Rate, k
18.4 days	

$\frac{1}{5}A_0 = A_0 = e^{K(18.4)}$

k = −.037671
~~(Round to six decimal places~~ as needed.)

$\ln(.5) = K(18.4) \Rightarrow K = \frac{\ln(.5)}{18.4}$

$K = -.037671$

K =

8. The half-life of a certain tranquilizer in the bloodstream is 28 hours. How long will it take for the drug to decay to 84% of the original dosage? Use the exponential decay model, $A = A_0 e^{kt}$, to solve.

First find K; $\frac{1}{2}A_0 = A_0 e^{K(28)}$ $\Rightarrow K = \frac{\ln(.5)}{28} = -.024755264$

7.0 hours
(Round to one decimal place as needed.)

Now: $.84A_0 = A_0 e^{-.024755t} \Rightarrow .84 = e^{-.024755t} \Rightarrow t = \frac{\ln(.84)}{-.024755} = 7.0$ ≈ 7

9. Use the exponential growth model, $A = A_0 e^{kt}$, to find the time it takes a population to multiply by eleven (to grow from A_0 to $11A_0$).

$11A_0 = A_0 e^{Kt} \Rightarrow 11 = e^{Kt} \Rightarrow \ln(11) = Kt \Rightarrow t = \frac{\ln(11)}{K}.$

$t = \frac{\ln 11}{K}$ (Simplify your answer.)

10. ~~The logistic~~ growth function at right describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.

$$f(t) = \frac{119{,}000}{1 + 5400e^{-t}}$$

a. How many people became ill with the flu when the epidemic began?
b. How many people were ill by the end of the fourth week?
c. What is the limiting size of the population that becomes ill?

a. The number of people initially infected is 22.
(Round to the nearest number of people.)

a) $f(0) = \frac{119000}{1+5400e^{-0}} = 22.0324 \approx 22$

b. The number of people infected after 4 weeks is 1191.
(Round to the nearest number of people.)

b) $f(4) = \frac{119000}{1+5400e^{-4}} = 1191.13 \approx 1191$

c. The limiting size of the infected popluation is 119000.
(Round to the nearest number of people.)

11. A logistic growth model for world population, f(x), in billions, x years after 1976 is $f(x) = \frac{12.57}{1 + 4.11e^{-0.026x}}$. According to this model, when will the world population be 8 billion?

According to this model, the world population will be 8 billion in 2052.
(Round to the nearest whole number as needed.)

76 years from 1976 is 2052

$$8 = \frac{12.57}{1+4.11e^{-.026x}} \Rightarrow 1+4.11e^{-.026x} = \frac{12.57}{8}$$

$$\Rightarrow 4.11e^{-.026x} = \frac{12.57}{8} - 1$$

$$\Rightarrow e^{-.026x} = \frac{\frac{12.57}{8} - 1}{4.11}$$

$$\Rightarrow -.026x = \ln\left(\frac{\frac{12.57}{8}-1}{4.11}\right) \Rightarrow x = \frac{\ln\left(\frac{\frac{12.57}{8}-1}{4.11}\right)}{-.026}$$

$$x = 75.8981 \approx 76 \text{ years}$$

1. 361.8

2. 2014

3. 5

4. 24

   12

   6

   3

   1.5

5. 11,133

6. 9.5

7. -0.037671

8. 7.0

9. $\dfrac{\ln 11}{k}$

10. 22

    1191

    119,000

11. 2052